Volume: 4 **Issue: 8** **Pages: 37-39**

Prime Labeling of House Graphs

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Article Info.
E-ISSN: 2583-6528
Impact Factor (SJIF): 6.876
Peer Reviewed Journal
Available online:
www.alladvancejournal.com

Received: 12/July/2025
Accepted: 13/Aug/2025

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Abstract

Consider G = (M,N) be a graph with Ⱪ points and U lines is said to be prime labeling if the points are labeled with distinct integers that do not exceed 'Ⱪ', so that each pair of neighboring points a and b are relatively Prime. Prime labeling is an important area in Graph theory, as it connects number theory with Graph structures and provides arithmetic properties. In this paper, we focus on the study of Prime labeling for different families of Graphs. In particular, we examine the Prime Labeling of House graph and its denoted by HS. Various observations, characterization, and results are discussed to highlight how prime labeling can be established for these graph classes.

Keywords: Cycle (C_n), Path (P_n), House Graph (HS), Prime Labeling (*PL*).

Introduction

Consider the graph G = (M,N) are only finite, simple, and un-directed graphs. The point set and the line set are represented by M(G) and N(G). The terms 'Ⱪ' and 'U' stand for the graph G's order and size. Labeling of a graph means the points and lines are to assigned to the integers with certain conditions. We refer to J.A. Bondy and Murthy [1] for all other graph theory terms and notations. J.A. Gallian provides a thorough overview of graph labeling [2]. S.K.Vaidya and K.K. Kanani [6] have proved certain cycle related Graphs.

Definition 1.1

Consider G = (M,N) be a graph including ' Ⱪ ' points. A bijection mapping τ :M(G) → {1,2,3,…,Ⱪ } is defined as a PL if, for each line u= ij the labels assigned to the points i and j are relatively Prime. A graph that permits PL is referred to as a Prime graph (PG).

Definition 1.2

There are 'n ' vertices and 'n-1' edges in the path P_n.

Definition 1.3

The Cycle Graph C_n is a non-empty trail in which only the first and last vertices are equal.

Definition 1.4

House Graph is a simple graph with five points and six lines where the vertices represent the corners of the house and the lines represents the connections between them. Additionally, it has a square base with four points and one additional point on top connected to two opposite corners of the square shows that a roof. It is denoted by HS. And also Base extension of the House graph is denoted as $HS(B_n)$. and Roof extension of the House graph is denoted as $HS(R_n)$.

Main Results

Theorem 2.1

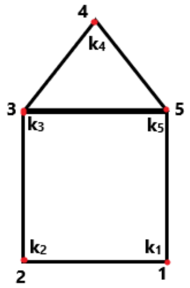
The Single House Graph HS is a PG.

Proof: Let G be a House graph HS with five points.

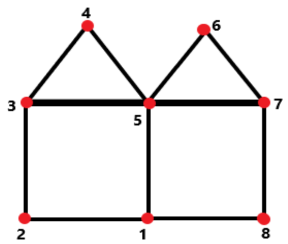
Take M(G)= $\{Ⱪ_1, Ⱪ_2, Ⱪ_3, Ⱪ_4, Ⱪ_5\}$.

Therefore |Ⱪ | = 5.

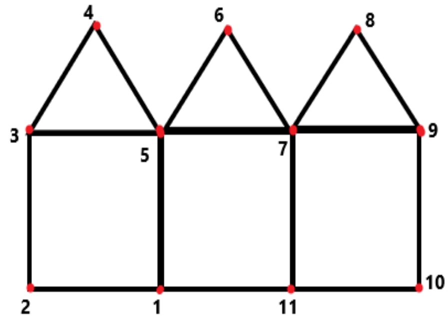
Define a Mapping τ: M(G) $\longrightarrow$ { 1,2,…,5 } by
τ (K_a) = a ; $1 \leq a \leq 5$.
Based on this pattern,
Gcd {τ(K_3), τ(K_5)} = 1
Gcd {τ(K_a), τ(K_{a+1})} = 1 ; $1 \leq a \leq 4$
Gcd {τ(K_5), τ(K_1)} = 1.
Therefore for every line u = ij where i and j are relatively Prime. Therefore the HS is a PG.
Example: Prime Labeling of HS



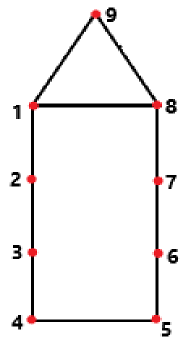
Theorem 2.2
The Double adjacent House graph 2(HS) is a PG.
Proof: Let G be a Double House graph 2(HS).
Take M(G)= { K_1, K_2, K_3, …., K_8}.
Therefore |K| = 8.
Define a Mapping τ: M(G) $\longrightarrow$ { 1,2,…,8 } by
τ (K_a) = a ; $1 \leq a \leq 8$.
Based on this pattern,
Gcd { τ (K_3), τ (K_5)} = 1
Gcd {τ(K_a), τ(K_{a+1})} = 1 ; $1 \leq a \leq 7$
Gcd {τ(K_5), τ(K_1)} = 1
Gcd {τ(K_5), τ(K_7)} = 1
Gcd {τ(K_8), τ(K_1)} = 1.
Therefore for every line u = ij where i and j are co-prime. Therefore the 2(HS) is a PG.
Example: Prime Labeling of 2(HS)



Theorem 2.3
The Trible adjacent House graph 3(HS) is a PG.
Proof: Let G be a Trible House graph 3(HS).
Take M(G)= {K_1, K_2, K_3, …., K_{11}}.
Therefore |K| = 11.
Define a Mapping τ: M(G) $\longrightarrow$ { 1,2,3,.….,11 } by
τ(K_a) = a ; $1 \leq a \leq 11$.
Based on this pattern,
Gcd {τ(K_3), τ (K_5)} = 1
Gcd {τ(K_a), τ(K_{a+1})} = 1 ; $1 \leq a \leq 10$
Gcd {τ(K_5), τ(K_1)} = 1
Gcd {τ(K_5), τ(K_7)} = 1
Gcd {τ(K_7), τ (K_9)} = 1
Gcd {τ(K_7), τ(K_{11})} = 1
Gcd {τ(K_{11}), τ(K_1)} = 1
Therefore for every line u = ij where i and j are co-prime. Therefore the 3(HS) is a PG.
Example: Prime Labeling of 3(HS)

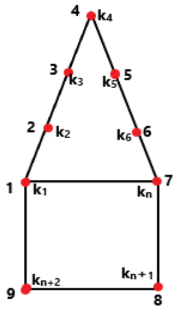


Theorem 2.4
Base extension of the House graph HS(B_n) is a PG.
Proof: Let G be a Base extension of the House graph HS(B_n), n>4 for all n.
Take M(G)= { K_1, K_2, K_3, …., K_n,K_{n+1}}.
Therefore |K| = n+1.
Define a Mapping τ: M(G) $\longrightarrow$ { 1,2,3,..….,n,n+1 } by
τ (K_a) = a ; $1 \leq a \leq n+1$
Based on this pattern,
Gcd {τ(K_1), τ(K_n)} = 1
Gcd {τ(K_n), τ(K_{n+1})} = 1
Gcd {τ(K_{n+1}), τ(K_1)} = 1
Gcd {τ(K_a), τ(K_{a+1})} = 1 ; $1 \leq a \leq 7$
Therefore for every line u = ij where i and j are co-prime. Therefore the HS(Bn) is a PG.
Example: Prime Labeling of Base extension of House Graph HS(B_8).



Theorem 2.5
The Roof extension of HS graph HS(R_n) is a PG.

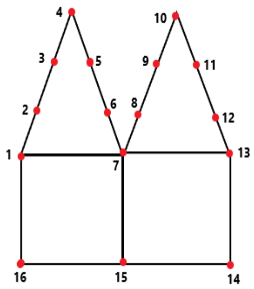
Proof: Let G be a Roof extension of House graph HS(R_n).
Take M(G)= {K_1, K_2, K_3, …., K_n,K_{n+1}, K_{n+2}}.
Therefore |K| = n+2
Define a Mapping τ: M(G) $\longrightarrow$ { 1,2,3,..….,n,n+1,n+2} by
τ(K_a) = a ; $1 \leq a \leq n$.
τ(K_{a+1}) = a ; $1 \leq a \leq n+1$
τ(K_{a+1}) = a ; $1 \leq a \leq n+2$
Based on this pattern,
Gcd {τ(K_n), τ(K_{n+1})} = 1
Gcd {τ(K_{n+1}), τ(K_{n+2})} = 1
Gcd {τ(K_1), τ(K_{n+2})} = 1
Gcd {τ(K_1), τ(K_n)} = 1
Gcd {τ(K_a), τ(K_{a+1})} = 1 ; $1 \leq a \leq n-1$.
Therefore for every line u = ij where i and j are co-prime. Therefore HS(R_n) is a PG
Example: Prime Labeling of Roof extension of House graph HS(R_7).



Theorem 2.6: The Roof extension of double house graph $2(HS(R_n))$ is a PG.

Proof: Let G be a Roof extension of Double House graph $2HS(R_n)$
Take $M(G)= \{ K_1, K_2, K_3, \ldots, K_7, K_8, \ldots, K_{15}, K_{16}\}$.
Therefore $|K| = 16$.
Define a Mapping $\tau: M(G) \longrightarrow \{1,2,3,\ldots,16\}$ by
$\tau(K_a) = a$; $1 \leq a \leq 16$.
Based on this pattern,
Gcd $\{\tau(K_1), \tau(K_7)\} = 1$
Gcd $\{\tau(K_1), \tau(K_{16})\} = 1$
Gcd $\{\tau(K_7), \tau(K_{13})\} = 1$
Gcd $\{\tau(K_7), \tau(K_{15})\} = 1$
Gcd $\{\tau(K_a), \tau(K_{a+1})\} = 1$; $1\leq a \leq 15$
Therefore for every line u = ij where i and j are relatively Prime. Therefore $2HS(R_n)$ is a PG.

Example: Prime Labeling of Roof of extension Double House graph $2HS(R_n)$.



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