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Vertex Prime Labeling of the Two Copies of Cycle Graphs

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Abstract
A graph with 'h' points and 't' lines $\mu_1 = (M_1, N_1)$ is said to have a vertex prime labeling (VPL) if its lines can be labeled by unique integers from $\{1,2,3,\ldots|N_1|\}$, such that for every degree at least two, the greatest common divisor of the labels on its event lines one. A graph that admits VPL is called a vertex prime graph (VPG). Graph labeling is an important area of research in Graph theory. There are many kinds of graph labeling such as Graceful labeling, Magic labeling, Prime labeling, and other differend labeling techniques. In In this work, we examine the VPG of two copies of Cycle ($2C_q$).

Keywords: Prime labeling , vertex prime labeling, Cycle and Path.

Introduction

In this paper we consider only the defined simple undirected graph. The graph μ_1 has a point set $M_1 = M_1(\mu_1)$ and line set $N_1=N_1(\mu_1)$. References and glossary are provided by Pandi and Murthy [3].The concept of prime labeling was introduced by Roger Entringer and discussed in a paper by Tout.A [2]. Meena.S and Vaithilingam.K [7] demonstrated that the and Helm H_h graph is a prime graph [9]. Meena.S and Ezhil.A [8] demonstrated that the Cycle C_h and path P_h graph are a total prime graph. We refer to Gallian. J.A. [1] as a current dynamic graph labeling examine.

Two integers' b and c are said to be relativity prime if the greatest common divisor is one. Relativity prime numbers play an important role in both analytic and algebraic number theory. Many researchers have studied prime graphs. For example Fu. H [5] have demonstrated that path P_h on h points is a prime graph. Karunakaran.V and Thenmozhi.S [9] have demonstrated that the Cycle C_h is a prime graph.

Definition: 1.1

Let $\mu_1 = (M_1(\mu_1),N_1(\mu_1))$ be a graph with h points and t lines. If for every line e = bc, Gcd $\{b_1(b),b_1(c)\} = 1$, then the bijective function $b_1 : M_1(\mu_1) \rightarrow \{1,2,\ldots,h\}$ is called a PL. A graph that admits a PL is called a PG.

Definition: 1.2

Let $\mu_1 = (M_1(\mu_1),N_1(\mu_1))$ be a graph with h points and t lines. The bijective function

b_1: $N_1(\mu_1) \rightarrow \{1,2,\ldots,t\}$ is called a VPL if every point has at least two degrees and the greatest common divisor of the labels on its event line is one. That is Gcd $\{b_1(b), b_1(c)\}=1$.

Definition: 1.3

Simple graph with h points ($h \geq 3$) and h lines forming a cycle of length h is called a cyclic graph C_h.

Definition: 1.4

The path P_h has h points and h-1lines.

Main Results: 2

Theorem: 2.1

A VPG is made up of two copies of Cycle ($2C_h$) (for all h) connected by a single common point.

Proof:

Let μ_1 be a graph consisting of two copies of Cycle ($2C_h$) connected by a single common point.

Let $M_1(\mu_1)= \{u_1,u_2,u_3,\ldots u_h,u_{h+1},u_{h+2},\ldots u_{2h-1}\}$

Let $N_1(\mu_1) =\{u_tu_{t+1} / 1 \leq t \leq h-1\} \cup \{u_hu_1\} \cup \{u_1u_{h+1}\} \cup \{ u_tu_{t+1} /$

$h+1 \leq t \leq 2h-2\} \cup \{u_{2h-1}u_1\}$ and $|M_1(\mu_1)|=2h-1$, $|N_1(\mu_1)| = 2h$

A bijection b_1 is defined by $b_1: N_1(\mu_1) \rightarrow \{1, 2, 3, \ldots 2h\}$

such that $b_1(w_t) = t$; $1 \leq t \leq 2h$

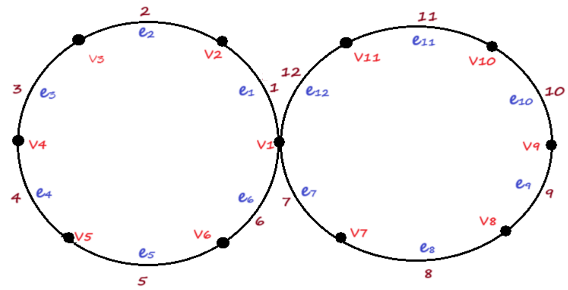
i) Gcd of the incident of all lines u_1 = Gcd $\{w_1, w_h, w_{h+1}, w_{2h}\}=1$

ii) Gcd of the incident of all lines u_t = Gcd $\{(w_{t-1}, w_t) / 2 \leq t \leq h\}=1$

iii) Gcd of the incident of all lines u_t= Gcd $\{(w_t, w_{t+1}) / h+1 \leq t \leq 2h-1\}=1$

Thus, the Gcd of each points of degree at least two all the event line is one.

Example: VPL of two copies of Cycle ($2C_6$)



Theorem: 2.2

A VPG is made up of two copies of Cycle ($2C_h$) (for all h) joining with two shared points.

Proof:

Let $M_1(\mu_1) = \{u_1, u_2, \ldots u_h, u_{h+1}, u_{h+2}, \ldots u_{2h-2}\}$

Let $N_1(\mu_1) = \{u_t u_{t+1} / 1 \leq t \leq h-1\} \cup \{u_1 u_h\} \cup \{u_t u_{t+1} / h \leq t \leq 2h-3\} \cup \{u_{2h-1} u_1\}$ and $|M_1(\mu_1)| = 2h-2$, $|N_1(\mu_1)| = 2h-1$.

A bijection b_1 is defined by $b_1 : N_1(\mu_1) \rightarrow \{1, 2, 3, \ldots (2h-1)\}$

Such that $b_1(w_t) = t$; $1 \leq t \leq 2h-1$

Theorem: 2.3

A VPG is made up of two copies of Cycle ($2C_h$) (h is even) connected by a single line.

Proof:

Let $M_1(\mu_1) = \{u_1, u_2, \ldots u_h, u_{h+1}, u_{h+2}, \ldots u_{2h}\}$

Let $N_1(\mu_1) = \{u_t u_{t+1} / 1 \leq t \leq h-1\} \cup \{u_1 u_h\} \cup \{u_1 u_{h+1}\} \cup \{u_t u_{t+1} / h+1 \leq t \leq 2h-1\} \cup \{u_{2h} u_{h+1}\}$ and $|M_1(\mu_1)|= 2h$, $|N_1(\mu_1)| = 2h+1$.

A bijection b_1 is defined by $b_1 : N_1(\mu_1) \rightarrow \{1, 2, 3, \ldots (2h+1)\}$

Such that $b_1(w_t) = t$; $1 \leq t \leq 2h+1$

Theorem: 2.4

A VPG is made up of two copies of Cycle ($2C_h$) (h is even) connected with two lines.

Proof:

Let $M_1(\mu_1) = \{u_1, u_2, \ldots u_h, u_{h+1}, u_{h+2}, \ldots u_{2h}\}$

Let $N_1(\mu_1) = \{u_t u_{t+1} / 1 \leq t \leq h-1\} \cup \{u_1 u_h\} \cup \{u_1 u_{h+1}\} \cup \{u_t u_{t+1} / h+1 \leq t \leq 2h-1\} \cup \{u_{2h} u_{h+1}\} \cup \{u_h u_{2h}\}$

and $|M_1(\mu_1)|= 2h$, $|N_1(\mu_1)| = 2h+2$.

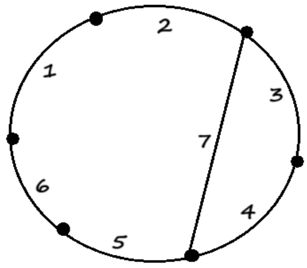
A bijection b_1 is defined by $b_1 : N_1(\mu_1) \rightarrow \{1, 2, 3, \ldots (2h+2)\}$

Such that $b_1(w_t) = t$; $1 \leq t \leq 2h+2$.

Theorem: 2.5

Cycle C_h (for all h) single chord which is joined by the points u_3 and u_5 is a VPG.

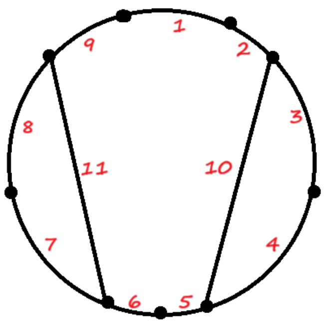
Example: Cycle C_6 single chord is a VPG.



Heorem: 2.6

Two chords of the Cycle C_h (for all h) is a VPG.

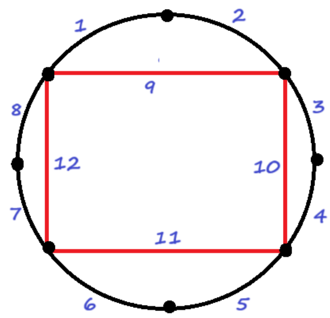
Example: Two chords of the Cycle C_9 is a VPG.



Theorem: 2.7

Cycle C_8 Square is a VPG.

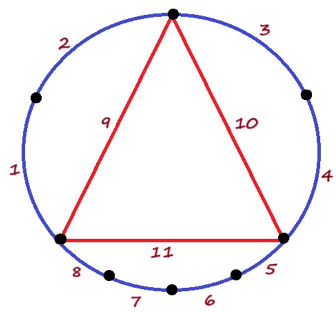
Example: Cycle C_8 square is a VPG.



Theorem: 2.8

Triangle of a Cycle C_8 is a VPG.

Example: Cycle C_8 is a VPG.



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