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Edge Prime Labeling of $M(C_n)$

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Abstract

A Graph G = (N,L) with ‘s’ nodes and ‘t’ lines is known as vertex prime labeling. A notation of vertex prime labeling(VPL). Line of the graphs are labeled from the set of natural numbers. If for every node of degree at least 2 the GCD(Greatest Common Divisor) of the labels on its incident line is 1. A vertex prime graph is a Graph G that allows vertex prime labeling. we discuss Multicycles are intersecting at a single node. In this paper the vertex prime labeling of Multicycle Graphs $M(C_n)$ is proved.

Keywords: Prime labeling (PL), Vertex Prime labeling (VPL), Multicycle Graph (M (C_n)), Path Graph (P_n), cycle (C_n) Graph.

Introduction

Consider only finite, simple and undirected graphs are taken here. G = (N, L) is a graph in which the node set represented by N(G) and the line set represented by L(G). The terms ‘s’,‘t’ stand for the graphs G’s order and size, respectively. The GCD (Greatest Common Divisor) of the labels on its incident lines is one. If the lines are labeled with ‘t’ natural numbers.

We already proved the PL of Multicycle graphs and now we have proved the VPL of the identical graphs.

We referred to J.A. Bondy and Morthy [1] for all other graph theory terms & symbols. Rojer Entringer created the PL notation, which was covered in a study by Tout [3]. Total Prime Labeling (TPL) of certain cycle and path Related Graphs have proved by S.Meena and A.Ezhil [14]. From the reference Mukund V. Bapat [5] have proved Vertex Prime Labeling of Path Related Graphs.

Definition 1.1:

Assume that G be a graph that has ‘s’ nodes and ‘t’ lines. A bijection $\emptyset$:L(G)→ {1,2, 3…(s)}, is referred to as VPL. The GCD of labels on the incident lines of each node of degree at least two is 1.

Definition 1.2:

Path P_n having ‘ŋ’ nodes and ‘ŋ -1’ lines.

Definition 1.3:

A Cycle graph is a simple graph containing ‘ŋ’ nodes (ŋ≥3) and ‘ŋ’ lines forming a cycle of Length ‘ŋ’.

Definition 1.4:

A graph that contains one or more cycles is considered a multicycle graph. It is denoted by M (C_n). Here ɱŋ represented by number of cycles and ŋ represented by cycle length.

Main Results:

Theorem 2.1:
Two Cycles $2(C_n)$ intersecting at a single node is a VPG.

Verification:
Consider G be a graph created by joining 2 cycles intersecting at a single node.

N(G) = $\{s_1, s_2, s_3, \ldots\ s_\eta, s_{\eta+1}, \ldots, s_{2\eta-1}\}$

&

L(G) = $\{s_i s_{i+1} / 1 \leq i \leq \eta - 1\} \cup \{s_1 s_\eta\} \cup \{s_1 s_{\eta+1}\} \cup \{s_i s_{i+1} / \eta + 1 \leq i \leq 2\eta - 2\} \cup \{s_1 s_{2\eta-1}\}$

The overall nodes s = 2η-1 and lines t = 2η

Mapping is defined by

$\emptyset : L(G) \longrightarrow \{1,2,3 \ldots (2\eta)\}$

$\emptyset(t_j) = j$ for $1 \leq j \leq 2\eta$

Based on this pattern,

GCD {every lines incident with t_1} = GCD {1, η, η+1,2η} = 1

GCD {every adjacent line incident with t_1} = GCD {j -1, j} = 1 for $2 \leq j \leq \eta$

GCD {every adjacent line incident with t_1} = GCD {j, j+1} = 1 for $\eta + 1 \leq j \leq 2\eta - 1$

Consequently every incident lines have a GCD that is 1, associated with every node of degree at least 2.
Therefore Graph $2(C_n)$ is a VPG.

Theorem 2.2:
Three Cycles $3(C_n)$ intersecting at a single node is a VPG.

Verification:
Consider G be a graph created by joining 3 cycles intersecting at a single node.

N(G) = $\{s_1, s_2, s_3, \ldots\ s_\eta, s_{\eta+1}, \ldots, s_{2\eta-1}, s_{2\eta}, s_{2\eta+1}, \ldots s_{3\eta-2}\}$

&

L(G) = $\{s_i s_{i+1} / 1 \leq i \leq \eta - 1\} \cup \{s_1 s_\eta\} \cup \{s_1 s_{\eta+1}\} \cup \{s_i s_{i+1} / \eta + 1 \leq i \leq 2\eta - 2\} \cup \{s_1 s_{2\eta-1}\} \cup \{s_1 s_{2\eta}\} \cup \{s_i s_{i+1} / 2\eta \leq i \leq 3\eta - 3\} \cup \{s_1 s_{3\eta-2}\}$

The overall nodes s = 3η-2 and lines t = 3η

Mapping is defined by

$\emptyset : L(G) \longrightarrow \{1,2,3 \ldots (3\eta)\}$

$\emptyset(t_j) = j$ for $1 \leq j \leq 3\eta$

Based on this pattern,

GCD{every lines incident with t_1} = GCD {1, η, η +1, 2η, 2η +1, 3η} = 1

GCD {every adjacent line incident with t_1} = GCD {j -1, j} =1 for $2 \leq j \leq \eta$

GCD {every adjacent line incident with t_1} = GCD {j, j+1} = 1 for $\eta + 1 \leq j \leq 2\eta - 1$

GCD {every adjacent line incident with t_1} = GCD {j +1, j+2} = 1 for $2\eta \leq j \leq 3\eta - 2$

Consequently every incident lines have a GCD that is 1, associated with every node of degree at least 2.
Therefore Graph $3(C_n)$ is a VPG.

Theorem 2.3:
Four Cycles $4(C_n)$ intersecting at a single node is a VPG.

Verification:
Suppose G be a graph created by joining 4 cycles intersecting at a single node.

N(G) = $\{s_1, s_2, s_3, \ldots\ s_\eta, s_{\eta+1}, \ldots, s_{2\eta-1}, s_{2\eta}, s_{2\eta+1}, \ldots s_{3\eta-2}, \ldots\ s_{4\eta-3}\}$

&

L(G) = $\{s_i s_{i+1} / 1 \leq i \leq \eta - 1\} \cup \{s_1 s_\eta\} \cup \{s_1 s_{\eta+1}\} \cup \{s_i s_{i+1} / \eta + 1 \leq i \leq 2\eta - 2\} \cup \{s_1 s_{2\eta-1}\} \cup \{s_1 s_{2\eta}\} \cup \{s_i s_{i+1} / 2\eta \leq i \leq 3\eta - 3\} \cup \{s_1 s_{3\eta-2}\} \cup \{s_1 s_{3\eta-1}\} \cup \{s_i s_{i+1} / 3\eta - 1 \leq i \leq 4\eta - 4\} \cup \{s_1 s_{4\eta-3}\}$

The overall nodes s = 4ŋ-3 and lines t = 4ŋ.
Mapping is defined by
$\emptyset$: L(G) ⟶ {1,2,3…. (4ŋ)}
$\emptyset$ ($t_{ǰ}$) = ǰ for 1≤ ǰ ≤ 4ŋ

Based on this pattern,
GCD{every lines incident with t_1} = GCD {1, ŋ, ŋ +1, 2ŋ, 2ŋ+1,3ŋ, 3ŋ+1,4ŋ} = 1
GCD {every adjacent line incident with t_1} = GCD {ǰ -1, ǰ} =1 for 2 ≤ ǰ ≤ ŋ
GCD{every adjacent line incident with t_1} = GCD {ǰ, ǰ +1} = 1 for ŋ+1 ≤ ǰ ≤ 2ŋ- 1
GCD {every adjacent line incident with t_1} = GCD {ǰ+1, ǰ +2} = 1 for 2ŋ ≤ ǰ ≤ 3ŋ- 2
GCD {every adjacent line incident with t_1} = GCD {ǰ+2, ǰ+3} = 1 for 3ŋ-1 ≤ ǰ ≤4ŋ- 3

Consequently every incident lines have a GCD that is 1, associated with every node of degree at least 2.
Therefore Graph 4(C_n) is a VPG.

Theorem 2.4:
Ϻ Cycles Ϻ (C_n) intersecting at a single node is a VPG.

Verification:
Consider G be a graph created by joining Ϻ cycles intersecting at a single node.
N(G) = {s_1, s_2, s_3,…… $s_ŋ$,$s_{ŋ+1}$,……, $s_{2ŋ-1}$,$s_{2ŋ}$, $s_{2ŋ+1}$,……$s_{3ŋ-2}$,……$s_{4ŋ-3}$,….$s_{ɱ(ŋ-1)+1}$}
&
L(G) ={$s_1 s_{(ɱ-1)ŋ-(ɱ-3)}$}∪{$s_i s_{i+1}$ /(ɱ -1) ŋ - (ɱ -3) ≤i ≤ ɱ (ŋ -1)} ∪{$s_1 s_{ɱŋ-(ɱ-1)}$}

The overall nodes s = ɱ (ŋ - 1) and lines t = ɱŋ.
Mapping is defined by
$\emptyset$: L(G) ⟶ {1,2,3…. (ɱŋ)}
$\emptyset$ ($t_{ǰ}$) = ǰ for 1≤ ǰ ≤ ɱŋ

Based on this pattern,

GCD {every lines incident with t_1} = GCD {1, ŋ, ŋ+1, 2ŋ, 2ŋ+1, 3ŋ, 3ŋ+1,4ŋ .... (ɱ -1)ŋ +1, ɱŋ} = 1
GCD {every adjacent line incident with t_1} = GCD {ǰ -1, ǰ} = 1 for 2 ≤ ǰ ≤ ŋ
GCD {every adjacent line incident with t_1} = GCD {ǰ, ǰ +1} = 1 for ŋ+1 ≤ ǰ ≤ 2ŋ - 1
GCD {every adjacent line incident with t_1} = GCD {ǰ +1, ǰ +2} = 1 for 2ŋ ≤ ǰ ≤ 3ŋ - 2
GCD {every adjacent line incident with t_1} = GCD {ǰ +2, ǰ +3} = 1 for 3ŋ-1 ≤ ǰ ≤ 4ŋ – 3

Generally
GCD {every adjacent line incident with $t_{ǰ}$} = GCD {ǰ+(ɱ -2), ǰ+(ɱ -1)} =1 for ɱ (ŋ -1)-ŋ +3 ≤ ǰ ≤ ŋ+(ɱ -1)(ŋ-1)

Consequently every incident lines have a GCD that is 1, associated with every node of degree at least 2.
Therefore Graph Ϻ (C_n) is a VPG.

Theorem 2.5:
Ϻ Cycles Ϻ (C_n) intersecting at a single node @ P_1 is a VPG.

Verification:
Consider G be a graph created by joining Ϻ cycles intersecting at a single node @ P_1.
N(G) = {s_1, s_2, s_3,… $s_ŋ$,$s_{ŋ+1}$,…, $s_{2ŋ-1}$,$s_{2ŋ}$, $s_{2ŋ+1}$,….$s_{3ŋ-2}$,….$s_{4ŋ-3}$,….$s_{ɱ(ŋ-1)+1}$,$s_{ɱ(ŋ-1)+2}$}
&
L(G)={ $s_1 s_{(ɱ-1)ŋ-(ɱ-3)}$}∪{$s_i s_{i+1}$ /(ɱ -1)ŋ-(ɱ -3) ≤i ≤ ɱ (ŋ-1)} ∪{ $s_1 s_{ɱŋ-(ɱ-1)}$}∪{$s_1 s_{(ŋ+(ɱ-1)(ŋ-1))+1}$}

The overall nodes s = ɱ (ŋ-1)+2 and lines t = ɱ ŋ+1
Mapping is defined by
$\emptyset$: L(G) ⟶ {1,2,3…. (ɱŋ+1)}
$\emptyset$ ($t_{ǰ}$) = ǰ for 1≤ǰ ≤ɱŋ+1

Based on this pattern,

GCD {every lines incident with t_1} = GCD {1, ŋ, ŋ+1, 2ŋ, 2ŋ +1, 3ŋ, 3ŋ+1, 4ŋ,.... (ɱ -1)ŋ +1, ɱŋ, ɱŋ +1} = 1

GCD {every adjacent line incident with t_1} = GCD {ǰ-1, ǰ} = 1 for 2 ≤ ǰ ≤ ŋ

GCD {every adjacent line incident with t_1} = GCD {ǰ, ǰ+1} = 1 for ŋ+1 ≤ ǰ ≤ 2ŋ -1

GCD {every adjacent line incident with t_1} = GCD {ǰ+1, ǰ+2} = 1 for 2ŋ ≤ ǰ ≤ 3ŋ -2

GCD {every adjacent line incident with t_1} = GCD {ǰ+2, ǰ+3} = 1 for 3ŋ-1 ≤ ǰ ≤ 4ŋ – 3

Generally

GCD {every adjacent line incident with $t_{\check{j}}$} = GCD {ǰ+(ɱ -2), ǰ+(ɱ -1)} = 1 for ɱ (ŋ-1)-ŋ+3 ≤ ǰ ≤ ŋ+(ɱ -1)(ŋ-1)

GCD {every adjacent line incident with $t_{\check{j}}$} = GCD {ǰ+(ɱ -2), ǰ+(ɱ -1)} = 1 for ɱ (ŋ-1)-ŋ+3 ≤ ǰ ≤ (ŋ+(ɱ -1)(ŋ-1)+1)

Consequently every incident lines have a GCD that is 1, associated with every node of degree at least 2.
Therefore Graph M (C_n) @ P_1 is a VPG.

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