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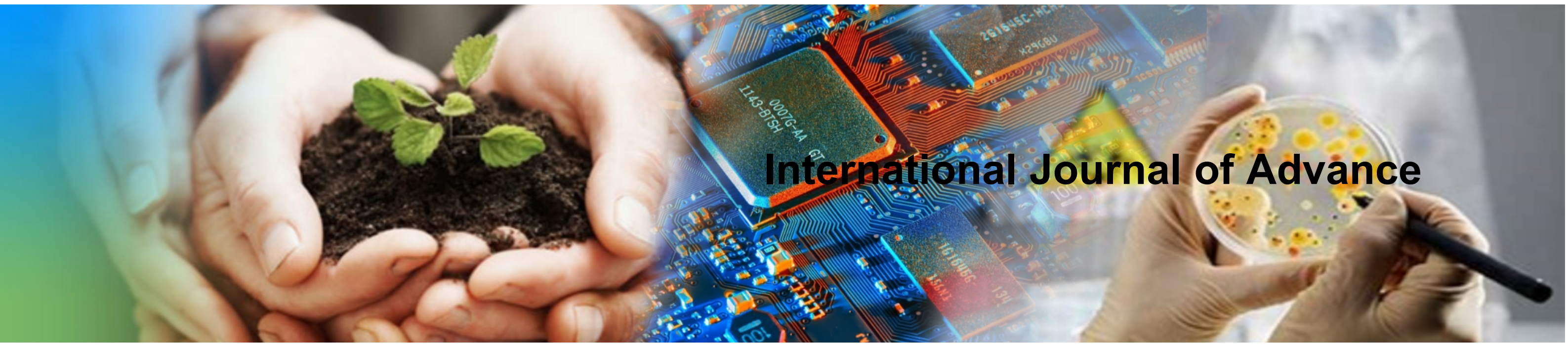

Prime Labeling of Two Copies of Cycle Related Graphs

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Abstract

If a graph Ǥ = (V, E) with 'r' vertices, and its vertices are named by unique positive integers not greater than n, then every pair of adjacent vertices is relatively prime. A graph that admits prime numbers is called a prime graph (PG).Graph labeling (GL) is an important area of research in Graph theory. There are many kinds of graph labeling such as Graceful labeling, Magic labeling, Prime labeling, and other differend labeling techniques. In this paper, we discuss the primes of two copies of a Cyclic graph (CG). We also discuss the primes of some graph functions and the paths between two graphs.

Keywords: Prime labeling, Cycle, Path.

Introduction

In this paper, we consider only a finitely simple undirected graph. The graph Ǥ has a set of vertices V = V(Ǥ) and an edge set E = E(Ǥ). References and terminology are given by Pandy and Moorthy [6].

The concept of prime labeling (PL) was introduced by Roger Entringer and discussed in a paper by Dowd.A (1982 P 365-368) [2].

Tereski.D (1991 P 359-369) [3] proved that the cyclic graph Cr on r vertices is a PG. Lee.S (1998 P 59-67) [4] proved that n is a PG while Meena.S and Vaithilingam.) K proved that the Helm Hn graph is a PG (2013 P 1075-1085) [9]. Meena.S and Ezhil.A proved that the cyclic Cr and path Pr graph is a total prime graph (2019 P 685-693) [10]. We refer to [1] Kalyan. J.A. (2009) for a recent dynamic survey on graph labeling.

Two integers a and b are said to be relatively prime if their greatest common divisor is one. Relative primes play an important role in both analysis and algebraic number theory. Many researchers have studied PG. For example, Fu. H (1994 P 181-186) [5] proved that the path Pr on r vertices is a PG.

Definition: 1.1

Labeling a graph is the process of assigning integers to vertices or edges, or both, subject to certain conditions.

Definition: 1.2

Let Ǥ = (V(Ǥ),E(Ǥ)) be a graph with 'r' vertices and 'd' edges. If for every edge e=uv, Gcd{ʃ(u) , ʃ(v)}=1, then the isosymmetric ʃ:V(Ǥ)→{1,2,…..,c}is called a prime labeling (PL).A graph that admits a PL is called a prime Graph (PG).

Definition: 1.3

A simple graph with r vertices ($r \geq 3$) and d edges that forms a cycle of length r is called a Cr Cycle graph (CG).

Definition: 1.4

The path P_r has r vertices and r-1 edges.

Main Results: 2

Theorem: 2.1

Two copies of CG Cr joining a single common vertex (for all r) is a PG.

Proof:

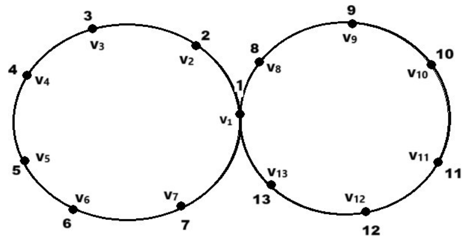
Let Ǥ be a graph consisting two copies of CG Cr joining a single common vertex. Let V(Ǥ) = { $v_k v_{k+1}$ ⁄ $1 \leq k \leq r-1$ } U { $v_r v_1$ } U { $v_1 v_{r+1}$ } U{ $v_k v_{k+1}$ ⁄ $r+1 \leq k \leq 2r-2$}U { $v_{2r-1} v_1$ } and |V(Ǥ) | = 2r-1

A bijection ʃis defined by ʃ:V(Ǥ) → {1, 2, 3,…..(2r-1)} such that ʃ(vk) = k ; $1 \leq k \leq 2r-1$

i) Gcd { ʃ(vk), ʃ(vk+1)⁄ $1 \leq k \leq r-1$} = 1

ii) Gcd { ʃ(vr) , ʃ(v1) } = 1
iii) Gcd { ʃ(vk) , ʃ(vk+1)/ r+1 ≤ k ≤ 2r-2} = 1 iv) Gcd { ʃ(v2r-1) , ʃ(v1) } = 1

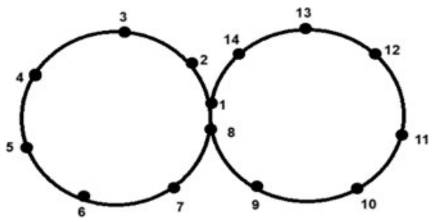
Hence ʃ agrees with the PL.
Therefore G is PG.
Example: PL of two copies of CG C7



Theorem: 2.2
Two copies of CG Cr joining with two common vertices (for all r) is a PG.

Proof:
Let G be a graph with two copies of CG Cr joining with two common vertices. Let V(G) = { vkvk+1 / 1 ≤ k ≤ r-1 } U { v1vr } U { vkvk+1 / r ≤ k ≤ 2r-3 } U { v2r-2v1 } and |V(G)| = 2r-2.
A bijection ʃ is defined by ʃ: V(G) → { 1,2,3,…..(2r-2) } such tha ʃ(vk) = k ; 1 ≤ k ≤ 2r-2
i) Gcd { ʃ(vk) , ʃ(vk+1) / 1 ≤ k ≤ r-1 } = 1
ii) Gcd { ʃ(v1) , ʃ(vr) } = 1
iii) Gcd { ʃ(vk) , ʃ(vk+1) / r ≤ k ≤ 2r-3 } = 1 iv) Gcd { ʃ(v2r-2) , ʃ(v1) } = 1

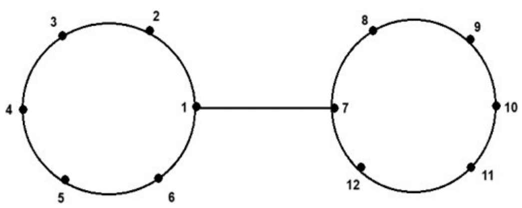
Hence ʃ agrees with the PL. Therefore G is PG.
Example: PG of two copies of CG C8



Theorem: 2.3
Two Copies of CG Cr joining a single edge (for all r , r is even) is a PG.

Proof:
Let G be a graph with two copies of CG Cr joining a single common edge.
Let V(G) ={ vkvk+1 / 1 ≤ k ≤ r-1} U { vrv1 } U { v1vr+1 } U { vkvk+1 / r+1 ≤ k ≤ 2r-1 } U { v2rvr+1 } and |V(G)| = 2r
A bijection ʃ is defined by ʃ : V(G) → {1,2,3,…….2r} such that ʃ (vk) = k ; 1 ≤ k ≤ 2r
i) Gcd {ʃ (vk) , ʃ (vk+1) / 1 ≤ k ≤ r-1} = 1
ii) Gcd { ʃ (vr) , ʃ (v1) } = 1
iii) Gcd { ʃ (v1) , ʃ (vr+1) } = 1
iv) Gcd { ʃ (vk) , ʃ(vk+1) / r+1 ≤ k ≤ 2r-1 } = 1 v) Gcd { ʃ (v2r) , ʃ (vr+1) } = 1

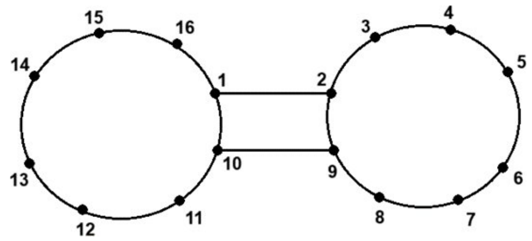
Hence ʃ agrees with the PL Therefore G is PG.
Example: PG of two copies of CG C6.



Theorem: 2.4
Two Copies of CG Cr joining with two edges (for all r , r is even) is a PG.

Proof:
Let G be a graph with two copies of CG Cr joining two edges.
Let V(G) = { vkvk+1 / 2 ≤ k ≤ r-1 } U { vr+1v2 } U { v2rv1}U{vr+1vr+2}U{v1v2}U{v1vr+2}U{vkvk+1/ r+2 ≤ k ≤ 2r-1} and |V(G)|=2r.
A bijection ʃ is defined by ʃ : V(G) → { 1,2,3,……2r } such that ʃ (vk) = k ; 1 ≤ k ≤ 2r.
i) Gcd { ʃ (vk) ,ʃ (vk+1) / 2 ≤ k ≤ r-1 } = 1
ii) Gcd { ʃ (vr+1) , ʃ (v2) } = 1
iii) Gcd { ʃ (v2r) , ʃ (v1) } = 1 iv) Gcd { ʃ(vr+1),ʃ(vr+2) } =1 v) Gcd { ʃ (v1),ʃ(v2)} =1
iv) Gcd { ʃ(v1),ʃ(vr+2)}=1
v) Gcd { ʃ(vk),ʃ(vk+1)/ r+2 ≤ k ≤ 2r-1}=1 Hence ʃ agrees with the PL.

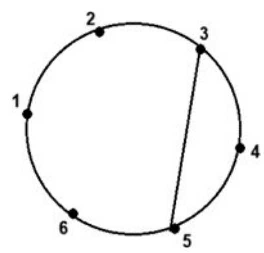
Therefore G is PG.
Example: PG of two copies of CG C8.



Theorem: 2.5
Chord (single) of CG Cr (for all r) is a PG.

Proof:
Let G be a CG Cr with single chord.
Let V(G) = {vkvk+1 / 1 ≤ k ≤ r-1} U {v3v5} and |V(G)| = r.
A bijection ʃ is defined by ʃ: V (G) → { 1,2,3,…..r } such that ʃ(vk) = k ; 1 ≤ k ≤ r.
i) Gcd { ʃ(vk) , ʃ (vk+1) / 1 ≤ k ≤ r-1 } = 1
ii) Gcd { ʃ(vr) , ʃ(v1) } = 1
iii) Gcd { f(vr) , ʃ (v5) } = 1 Hence ʃ agrees with the PL

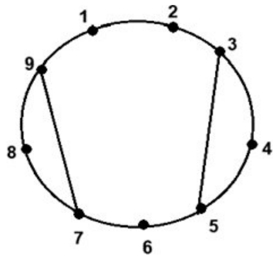
Therefore G is PG.
Example: PG of chord of CG C6.



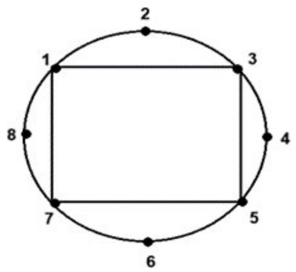
Theorem: 2.6
Chord (two) of CG Cr (for all r) is a PG.

Proof:
Let G be a CG Cr with double chord.
Let V(G) = { vkvk+1 / 1 ≤ k ≤ r-1} U {vrv1} U {v3v5} U {v7v9} and |V(G)| = r.
A bijection ʃ is defined by ʃ : V (G) → { 1,2,3,…..r } such that ʃ (vk) = k ; 1 ≤ k ≤ r.
i) Gcd { ʃ (vk) , ʃ (vk+1) / 1 ≤ k ≤ r-1 } = 1
ii) Gcd { ʃ (Vr) ,ʃ (v1) } = 1
iii) Gcd { ʃ (v3) , ʃ (v5) } = 1 iv) Gcd { ʃ (v7) , ʃ (v9) }=1

Hence ⨍ agrees with the PL Therefore G is PG.
Example: PG of chord (two) of CG C9.



Theorem: 2.7
Square of CG C8 is a PG.
Proof:
Let G be a CG C8 which contains inside square it is obtained by joining four vertices by an edge randomly.
Let V(G) = { vkvk+1 / 1 ≤ k ≤ 7 } U { v2k-1v2k+1 / 1 ≤ k ≤ 3 } U { v7v1 } and |V(G)| = 8 A bijection ⨍ is defined by ⨍ : V (G) → {1,2,3,…..8} such that ⨍(vk) = k ; 1 ≤ k ≤ 8
i) Gcd {⨍(vk) , ⨍(vk+1) / 1 ≤ k ≤ 7 } = 1
ii) Gcd { ⨍(v2K-1) , ⨍(v2K+1) / 1 ≤ K ≤ 3 } = 1
iii) Gcd { ⨍(v7), ⨍(v1) } = 1 Hence ⨍agrees with the PL.
Therefore G is PG.
Example: PG of square of CG C8.



Theorem: 2.8
Triangle of CG is PG.
Proof:
Let G be a CG C8 which contains inside the triangle it is obtained by joining three vertices by an edge randomly.
Let V(G) = { vkvk+1 / 1 ≤ k ≤ 7 } U { v2k-1v2k+1 / 1 ≤ k ≤ 2} U { v5v1 } U { v8v1 } and |V(G)| = 8 A bijection ⨍ is defined by ⨍: V (G) → { 1,2,3,…..8 } such that ⨍(vk) = k ; 1 ≤ k ≤ 8
i) Gcd { ⨍(vk), ⨍(vk+1) / 1 ≤ k ≤ 7 } = 1
ii) Gcd { ⨍(v2k-1), ⨍(v2k+1) / 1 ≤ k ≤ 2 } = 1
iii) Gcd { ⨍(v8), ⨍(v1) } = 1 iv) Gcd { ⨍(v5), ⨍(v1) } = 1 Hence ⨍agrees with the PL.
Therefore G is PG.
Example: PG of triangle of CG C8.

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