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Prime Labeling of Multicycle Graphs

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Abstract

G= (V, E), a graph of 'P' vertices. and 'q' edges is said to have a prime labeling if pvertices. are labeled with distinct Positive integers 1,2, 3....**p** that do not exceed "p", so that reach pair of neighboring Vertices u and v are co-prime, A prime Graph (PG) is a graph G that admits prime labeling, graph labeling is an important area of research in Graph Theory (GT). There are many types of graphs labeling and other different labeling techniques.In this work, examine Whether Multicycle graphs M (Cń) have prime labeling We also discuss Prime Labeling in the context of some graph operations namely cycle and Path.

Keywords: Labeling, prime labeling, cycle graph, path graph.

Introduction

Prime Labeling:

Assume that a graph with "p" vertices is G = (V,E). A labelling f: V(G) {1,2,3,……p}, the labels given to the vertices u and v are substantially prime for each edge e = uv, then V(G) is considered to have prime labeling. Prime Graph (PG) is a graph that admits prime labeling.

Path:

Path $P_n = v_1v_2v_3$ ………..$v_{ń}$ has 'ń' vertices and 'ń -1' edges.

Cycle:

Cycle $C_n = v_1v_2v_3$ ………..$v_{ń}\ v_1$ has 'ń' vertices and 'ń' edges.

Main Results:

Theorem 2.1:

Two copies of Cycles intersecting vertex graph at a common vertex is a PG.

Proof: Let G be a graph obtained from the two copies cycles intersecting at a common vertex it is denoted by $2(C_n)$.

Let V(G) = $\{v_1, v_2, v_3, \ldots\ldots v_{ń}, v_{ń+1}, \ldots\ldots v_{2ń-1}\}$

The total number of vertices p is 2 -1.

The labelling is defined as f (V) {1, 2, ⟶..(2 ń - 1) } with $f(v_i) = i$ for $1 \le i \le 2ń - 1$

According to this pattern, the (ĝćđ is greatest common divisor)

ĝćđ{ $f(v_1), f(v_n)$} = 1
ĝćđ { $f(v_1), f(v_{ń+1})$} = 1
ĝćđ { f $v_1), f(v_{2ń-1})$} = 1
ĝćđ { $f(v_i), f(v_{j+1})$} = 1 for $1 \le j \le ń - 1$
ĝćđ { $f(v_i), f(v_{j+1})$} = 1 for $n+1 \le j \le 2ń - 2$

Consequently e = uv for each edge, having u and v as co primes.

Hence, graph $2(C_{ń})$ is a PG.

Theorem 2.2

Three copies of Cycles intersecting at a common vertex graph is a PG.

Proof: Let G be a graph obtained from 3(Cń) which is a common vertex where the three copies of cycles intersecting.

Let V(G) =
$\{v_1, v_2, v_3, \ldots\ldots v_{ń}, v_{ń+1}, \ldots\ v_{2ń-1}, v_{2ń}, v_{2ń+1}, \ldots v_{3ń-2}\}$

The total number of vertices p is 3 ń -2.
The labelling is defined as f (V) {1, 2, .⟶.(3 ń - 2)} with
f (v_i) = i for 1 ≤ i ≤ 3 ń -2

According to this pattern, the
ĝćđ { f (v_1), f ($v_{ń}$)} = 1
ĝćđ { f (v_1), f ($v_{ń+1}$)} = 1
ĝćđ { f (v_1), f ($v_{2ń-1}$)} = 1
ĝćđ { f (v_1), f ($v_{3ń-2}$)} = 1
ĝćđ { f (v_1), f ($v_{2ń}$)} = 1
ĝćđ { f (v_i), f (v_{j+1})} = 1 for 1 ≤ j ≤ ń -1
ĝćđ { f (v_i), f (v_{j+1})} = 1 for ń +1 ≤ j ≤ 2 ń -2
ĝćđ { f (v_i), f (v_{j+1})} = 1 for 2 ń ≤ j ≤ 3 ń -3
Consequently e = uv for each edge, having u and v as co primes.
Hence, graph 3(Cń) is a PG.

Theorem 2.3:
Four copies of Cycles intersecting at a common vertex graph is a PG.

Proof: Let G be a graph obtained from 4(Cń) which is a common vertex where the fourcopies of cycles intersecting.
Let V(G) = {v_1,v_2, v_3, …….$v_{ń}$, $v_{ń+1}$,…. $v_{2ń-1}$, $v_{2ń}$,$v_{2ń+1}$,…$v_{3ń-1}$,$v_{3ń}$,$v_{3ń-1}$,….$v_{4ń-3}$}

The total number of vertices p is 4 ń -3.
The labelling is defined as f (V) {1, 2, ⟶..(4 ń - 3)} with
f (v_j) = j for 1 ≤ j≤ 4 ń -3

According to this pattern, the
ĝćđ { f (v_1), f ($v_{ń}$)} = 1
ĝćđ { f (v_1), f ($v_{ń+1}$)} = 1
ĝćđ { f (v_1), f ($v_{2ń}$)} = 1
ĝćđ { f (v_1), f ($v_{3ń-1}$)} = 1
ĝćđ { f (v_1), f ($v_{2ń-1}$)} = 1
ĝćđ { f (v_1), f ($v_{3ń-2}$)} = 1
ĝćđ { f (v_1), f ($v_{4ń-3}$)} = 1
ĝćđ { f (v_i), f (v_{j+1})} = 1 for 1 ≤ j ≤ ń -1
ĝćđ { f (v_i), f (v_{j+1})} = 1 for n+1 ≤ j ≤ 2 ń -2
ĝćđ { f (v_i), f (v_{j+1})} = 1 for 2n+1 ≤ j ≤ 3 ń -3
ĝćđ { f (v_i), f (v_{j+1})} = 1 for 3n+1≤ j ≤ 4 ń -4
Consequently e = uv for each edge, having u and v as co primes.
Hence graph 4(C ń) is a PG.

Theorem 2.4:
M copies of Cycles intersecting at a common vertex graph is a PG.

Proof: Let G be a graph obtained from the M copies of cycles intersecting at a common vertex it is denoted by M(C ń).
Let V(G) = { v_1,v_2, v_3, …….$v_{ń}$, $v_{ń+1}$,…. $v_{2ń-1}$, $v_{2ń}$,$v_{2ń+1}$,…$v_{3ń-1}$, $v_{3ń}$,$v_{3ń-2}$,….$v_{4ń-3}$, ….$v_{ɱ(ń-1)+1}$}. The total number of vertices p is ɱ(ń -1)+1.

Define a labeling
f (V) ⟶{ 1, 2, 3, 4, ……..(ɱ (ń -1)+1) } by
f (v_j) = i for 1 ≤ j ≤ ɱ (ń -1)+1

According to this pattern, the
ĝćđ { f (v_1), f ($v_{ń}$)} = 1
ĝćđ { f (v_1), f (v_{n+1})} = 1
ĝćđ { f (v_1), f ($v_{2ń}$)} = 1
ĝćđ { f (v_1), f ($v_{3ń-1}$)} = 1
ĝćđ { f (v_1), f ($v_{2ń-1}$)} = 1
ĝćđ { f (v_1), f ($v_{3ń-2}$)} = 1
ĝćđ { f (v_1), f ($v_{4ń-3}$)} = 1

In general
ĝćđ { f (v_1), f ($v_{ɱ(ń-1)+1}$)} = 1
ĝćđ f (v_1), f ($v_{ɱ(ń-1)-ń+3}$)} = 1
g.c.d { f (v_j), f (v_{j+1})} = 1 for 1 ≤ j ≤ ń -1
ĝćđ {f (v_j), f (v_{j+1})} = 1 for ń +1 ≤ j ≤ 2 ń -2
ĝćđ { f (v_j), f (v_{j+1})} = 1 for 2 ń +1 ≤ j ≤ 3 ń -3
ĝćđ { f (v_j), f (v_{j+1})} = 1 for 3 ń +1≤ j ≤ 4 ń -4
ĝćđ { f (v_j), f (v_{j+1})} = 1 for (ɱ (ń -1)- ń +3 ≤ j ≤ ɱ (ń -1)
Consequently e = uv for each edge, having u and v as co primes
Hence, graph M(Cń) is a PG.

Theorem 2.5:
M copies of Cycles intersecting at a common vertex @ P_1 graph is a PG.

Proof: Let G be a graph obtained from the M copies of cycles intersecting at a common vertex @ P_1it is denoted by M(C ń)@P_1.
Let V(G) = {v_1,v_2, v_3, …….$v_{ń}$, $v_{ń+1}$,…. $v_{2ń-1}$, $v_{2ń}$,$v_{2ń+1}$,…$v_{3ń-1}$, $v_{3ń}$,$v_{3ń-2}$,….$v_{4ń-3}$, ………,,$v_{ɱ(ń-1)+1}$,$v_{ɱ(ń-1)+2}$}
ɱ (ń -1)+2 is total of vertices p.

The labelling is considered as
f (V)⟶{1, 2, 3, 4, ……..(ɱ (ń -1)+2) } by
f (v_j) = j for 1 ≤ j ≤ ɱ (ń -1)+2

According to this pattern, the
ĝćđ { f (v_1), f ($v_{ń}$)} = 1
ĝćđ { f (v_1), f ($v_{ń+1}$)} = 1
ĝćđ { f (v_1), f ($v_{2ń}$)} = 1
ĝćđ { f (v_1), f ($v_{3ń-1}$)} = 1
ĝćđ { f (v_1), f ($v_{2ń-1}$)} = 1
ĝćđ { f (v_1), f ($v_{3ń-2}$)} = 1
ĝćđ { f (v_1), f ($v_{4ń-3}$)} = 1

In general
ĝćđ { f (v_1), f ($v_{ɱ(ń-1)+1}$)} = 1
ĝćđ { f (v_1), f ($v_{ɱ(ń-1)+2}$)} = 1
ĝćđ { f (v_1), f ($v_{ɱ(ń-1)-n+3}$)} = 1
ĝćđ { f (vj_i), f (v_{j+1})} = 1 for 1 ≤ j ≤ ń -1
ĝćđ { f (v_j), f (v_{j+1})} = 1 for ń +1 ≤ j ≤ 2 ń -2
ĝćđ { f (v_j), f (v_{j+1})} = 1 for 2 ń +1 ≤ j ≤ 3 ń -3
ĝćđ { f (v_j), f (v_{j+1})} = 1 for 3 ń +1≤ j ≤ 4 ń -4
ĝćđ { f (v_j), f (v_{j+1})} = 1 for (ɱ (ń -1)- ń +3 ≤ j ≤ ɱ (ń -1)+1

Consequently e = uv for every edge, having u and v as co primes.
Hence, graph M(Cń)@P_1 is a PG.

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