Volume: 4 Issue: 4 Pages: 130-132

Detour Dominating Sets in Cartesian Product of Three Graphs

[*1] Y Sherlin Nisha, [2]V Rajeswari and [3]Y Merlin Nisha

[*1,2] Associate Professor, Department of Mathematics, Sri Sairam Institute of Technology, Chennai, Tamil Nadu, India.

[3] Assistant Professor, Department of Education, CSI College of Education, Parassala, Kerala, India.

Article Info.
E-ISSN: 2583-6528
Impact Factor (SJIF): 6.876
Peer Reviewed Journal
Available online:
www.alladvancejournal.com

Received: 21/March/2025
Accepted: 23/April/2025

***Corresponding Author**
Y Sherlin Nisha
Associate Professor, Department of Mathematics, Sri Sairam Institute of Technology, Chennai, Tamil Nadu, India.

Abstract
A dominating set for a graph *G* is a subset *S* of *V* (*G*) such that every vertex not in *S* is adjacent in *S* is at least any one vertex of *S*. In this paper presented the detour domination in Cartesian product of three graphs. In the Cartesian product of three graphs, we examine detour dominating sets, which are represented as G×H×K. We investigate the structural characteristics of these products and determine precise values and constraints for the detour domination number in a number of graph classes, such as paths, cycles, and complete graphs. Additionally, we look at how the detour domination number of the product graph is affected by the distinct characteristics of G×H×K. Our findings provide insights into how dominance parameters behave in more intricate graph structures, expanding on previous research on detour domination for single and double Cartesian products.

Keywords: Detour index, Domination number, maximum distance, elongated path.

Introduction

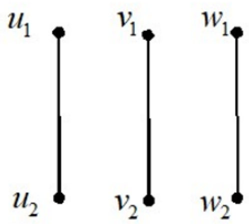
A graph G is a simple connected graph and finite. A graph G is a collection of points and lines represents vertices and edges of the graph denoted by V (G) and E(G). A set S of vertices in a connected graph G = (V, E) is called a detour set if every vertex not in S lies on a longest path between two vertices from S. A set S of vertices in G is called a dominating set of G if every vertex not in D has at least one neighbor in D A set S is a dominating set if every vertex u ∈ V − S lies on a longest path between two vertices from S.

A set S of vertices in G is called a detour dominating set of G if every vertex not in S has at least one neighbor in S. A detour dominating set D in G is a minimal detour dominating set if no proper subset of D is a dominating set. The minimum cardinality among all the minimal detour dominating set is called detour domination number of the graph G denoted by γD(G)

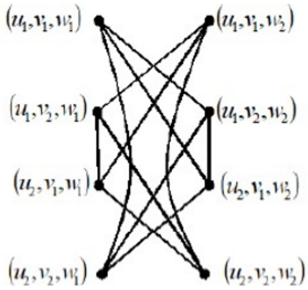
Definition: Cartesian product

Let G1(VG1, EG1) and G2(VG2, EG2) be two simple connected graphs. The Cartesian product of G1 and G2 denoted by G1 × G2 is a graph with vertex set VG1 × VG2, where two vertices (u1, v1) and (u2, v2) are adjacent if u1 = u2 and v1v2 ∈ E(G2) or v1 = v2 and u1u2 ∈ E(G1). This definition was further extented as second and third dimensional product of vertex measurable graphs. That is G1(VG1, EG1),G2(VG2, EG2) and G3(VG3, EG3) be three simple connected graph. The third dimensional product of vertex measurable graph of G1, G2 and G3 denoted by G1 × G2 × G3 is a graph V (G1) × V (G2) × V (G3), where two vertices (u1, v1, w1) and (u2, v2, w2) are adjacent if u1 = u2 and v1v2 ∈ E(G2)and w1w2 ∈ E(G3) or v1 = v2 and u1u2 ∈ E(G1)and w1w2 ∈ E(G3) or w1 = w2 and u1u2 ∈ E(G1)and v1v2 ∈ E(G2). In this paper we cover all the possibilities of connecting the vertices using the concept of Cartesian product of three graphs.

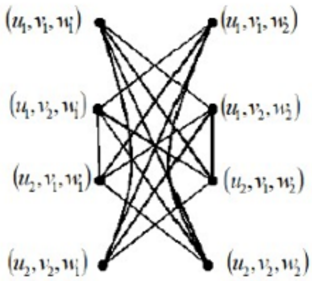
Consider the graphs $G_1 = G_2 = G_3 = K_2$. Then the Cartesian product of three graphs G_1, G_2 and G_3 is given by the following cases:



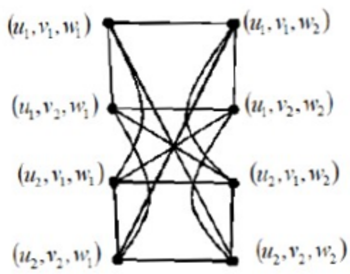
Case 1:



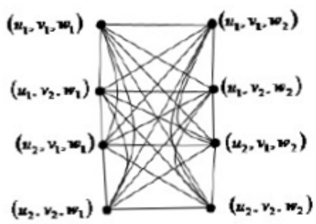
Case 6:



Case 7:

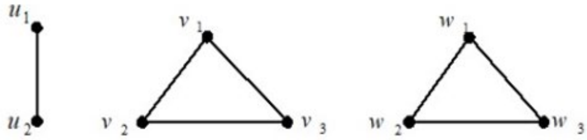


Case 9: The graph obtained is a complete graph.

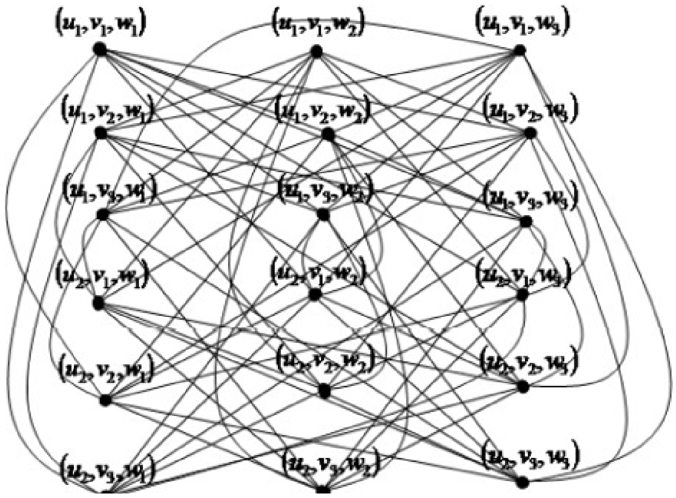


The graphs obtained in the above cases are non-isomorphic in nature. This shows that each case is a unique way of connecting the vertices given graphs G_1, G_2, G_3.

The concept of detour dominating set have been introduced in recent past by [3]. It is very interesting to investigate detour domination number of a graph as the detour domination numbers of very few graphs are known. Vaidya and Mehta [6] have derived detour domination number of degree splitting graph and helm graph while detour domination number of some cycle related graphs are discussed by Vaidya and Karkar [6]. The application of Cartesian product can be found in coding theory. Throughout this paper G means a finite, undirected graph without multiple edges or loops. Also Kn.Pn, Cn represents complete graphs paths and cycles with n vertices respectively.



Cartesian product of three complete graphs, $K_2 \times K_3 \times K_3$ in
by



References

1. Wiener H. Structural determination of paraffin boiling points, J. Am. Chem. Soc. 1947; 69:17-20.
2. Buckley F, Harary F. Distance in Graphs; Addison-Wesley: Reading, MA, 1990, 213-214.
3. Chartrand G, Haynes TW, Henning MA, Zhang P. Detour domination in graphs, Ars Combinatoria. 2004; 71:149-160.
4. John J, Arianayagam N. The detour domination number of a graph, Discrete Mathematics, Algorithms and Applications. 2017; 9(1).
5. Chartrand G, Johns GL, Tian S. Detour Distance in Grpahs, Annals of Discrete Math. 1993; 55:127-136.
6. Vaidya SK, Mehta RN. On detour domination in graphs, International Journal of Mathematics and Scientific Computing. 2016; 11:397-407.
7. Dankelmann P, Gutman I, Mukwembi S, Swart H. The edge-Wiener index of a graph, Discrete Mathematics. 2009; 309(10):3452-3457.
8. Chen S, Liu W. Extremal modified Schultz index of bicyclic graphs," MATCH. Communications in Mathematical and in Computer Chemistry. 2010; 64(3):767-782.
9. Chen S, Liu W. Extremal unicyclic graphs with respect to modified Schultz index, Utilitas Mathe-matica. 2011; 86(3):87-94.
10. Ivanciuc O, Balaban AT. Design of Topological Indices. Part 8. Path Matrices and Derived Molecular Graph Invariants, MATCH Commun. Math. Comput. Chem. 1994; 30:141-152.
11. Lukovits I. The Detour Index, Croat. Chem. Acta. 1996; 69:873-882.
12. Lukovits I. Indicators for Atoms Included in Cycles. J. Chem. Inf. Comput. Sci. 1996; 36:65-68.
13. Lukovits I, Razinger M. On calculation of the detour index, J. Chem. Inf. Comput. Sci. 1997; 37:283-286.