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A Study on Nonlinear Ion-Acoustic Solitary Waves in a Warm Dusty Plasma Considering Electron Inertia

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Abstract

The study focuses on the nonlinear wave structure of small-amplitude dust ion-acoustic solitary waves in a warm dusty plasma, in the presence of electron inertia. This investigation demonstrates the existence of both compressive and rarefactive solitons in a plasma model comprising negatively charged dust particles, electrons, and non-thermally distributed ions. To analyse these nonlinear ion-acoustic waves in a dusty plasma with electron inertia, the Korteweg-de Vries (KdV) equation is derived. It has been observed that the amplitude and width of compressive and rarefactive solitons respond differently to variations in pressure and the effects of electron inertia. The research outlines the conditions necessary for the existence of these nonlinear ion-acoustic solitons. The analysis is grounded in the fluid equations of motion that govern one-dimensional plasma. Different relational forms of the strength parameter (ε) are selected to stretch the space and time variables, resulting in various nonlinearities. These findings could have significant implications for astrophysical plasmas in inertial confinement fusion.

Keywords: Dusty plasma, Soliton, Ion-acoustic wave, Electron inertia, Perturbation theory.

Introduction

The study of ion-acoustic waves in plasma under various physical conditions has significantly impacted researchers since the early works of Korteweg and de Vries [1]. Many authors have theoretically explored non-linear ion-acoustic waves using the Korteweg-de Vries equation. Notably, Washimi and Taniuti [2] were the first to investigate the propagation of ion-acoustic solitary waves in cold plasma. Ikezi, Tailor, and Baker [3] were pioneers in experimentally discussing plasma ion-acoustic solitons and double layers.

Researchers such as Das and Tagare [4] and Das [5] have examined the effects of negative ions on solitons. They demonstrated that there is a critical density of negative ions at which solitary waves can become infinitely large. Kadomstev and Karpman [6] discussed the two phenomena of steepening and spreading, which contribute to the formation of solitary waves. Additionally, Sagdeev [7] investigated fully non-linear ion-acoustic solitary waves in an unmagnetised plasma containing hot electrons and cold ions, doing so without employing the reductive perturbation technique. Das [8], Watanabe [9], Verheest [10], and Baboolal *et al.* [11] have theoretically analyzed the existence of ion-acoustic solitons in plasmas that include negative ions. Kalita and Kalita [12] established the existence of modified Korteweg-de Vries solitons in various physical scenarios, while Tagare [13] also studied modified KdV solitons in isothermal conditions Electrons interact with warm positive and negative ions. Johnston and Epstein [14] studied the nonlinear ion-acoustic solitary waves in a cold, collisionless plasma through direct analysis of the field equation. They observed that even a very small change in the initial conditions could destroy the oscillatory behaviour of the solitary waves. Maitra and Roychoudhury [15] applied this technique to investigate dust acoustic solitary waves. Kalita and Barman [16] explored solitons in a warm, unmagnetized plasma, considering electron inertia and negative ions.

Recently, there has been a trend to investigate the characteristics of dusty plasma and its role in the formation of solitary waves and phenomena such as double layers. Many researchers have started to include dust particles in the study of space plasma. Stability analysis in dusty plasma is also an active area of research.

The creation of dusty plasma occurs due to the introduction of dust particles ranging from nano to one micrometre in size, which possess significant mass and support a variety of collective phenomena. These topics have recently garnered a large amount of research attention [17]. Charged dust grains cause many significant changes in the system's behaviour, including the creation of new modes, one of which is the dust acoustic wave [18].

Dusty plasma represents one of the most rapidly growing branches of plasma physics. In recent years, many publications in dusty plasma physics have focused on dust acoustic waves and dusty plasma lattices. By utilizing the reductive perturbation method, researchers have been able to analyze the propagation of dust acoustic waves.

The Korteweg-de Vries (KdV) equation has both mathematical and physical aspects. It can be solved exactly using analytical methods or through numerical techniques. Lin and Dun considered dust acoustic solitary waves in a dusty plasma with non-thermal ions [19]. The Kadomtsev-Petviashvili (KP) equation for dust acoustic waves in hot dusty plasmas was well established by Daun [20]. El-Labany and El Taibany [21] investigated dust-acoustic solitary waves and double layers in dusty plasma with an arbitrary streaming ion beam. Mahmood and Mushtaq [22] explored the effect of dust streaming on dust acoustic solitary waves in magnetized dusty plasmas. Gill *et al.* [23] studied dust acoustic solitary waves in a finite temperature dusty plasma with variable dust charge. Additionally, Aleksander Drenik and Richard Clergereaux [24] researched dusty plasma deposition of nanocomposite thin films, while Merlino *et al.* [25] examined nonlinear dust acoustic shocks and structures within the dusty plasma. With warm positive and negative ions Johnston and Epstein [14] studied the nonlinear ion acoustic solitary waves in a cold collisionless plasma by the direct analysis of the field equation. They observed that a very small change in the initial condition destroys the oscillatory behaviour of the solitary waves. Maitra and Roychoudhury [15] studied dust acoustic solitary waves using this technique Kalita and Barman [16] and Barman and Talukdar [26] have investigated solitons in a warm unmagnetized plasma with electron inertia and negative ions.

Recently, a trend has emerged to investigate the characteristics of dusty plasma and its role in the formation of solitary waves and double layers, among other phenomena. The inclusion of dust particles in the study of space plasma has gained traction among researchers. Stability analysis in dusty plasma has also become a significant area of investigation.

Dusty plasma is created by the introduction of dust particles that range from nano-sized to one micrometre. These particles carry significant mass and facilitate various collective phenomena, which have been the subject of numerous studies recently. The charged dust grains lead to substantial changes in system behaviour, including the emergence of new modes, such as the dust acoustic wave.

Dusty plasma represents one of the fastest-growing branches of plasma physics. In recent years, many published papers on dusty plasma physics have focused on dust acoustic waves and dusty plasma lattices. By employing the reductive perturbation method, we obtained insights into the propagation of dust acoustic waves.

The KdV equation has both mathematical and physical significance. It can be solved exactly through analytical methods or by using numerical techniques. Lin and Dun examined dust acoustic solitary waves in a dusty plasma containing non-thermal ions. The Kadomtsev-Petviashvili (KP) equation for dust acoustic waves in hot dust plasmas was well established by Daun WS. El-Labany and El Taibany investigated dust-acoustic solitary waves and double layers in a dusty plasma with an arbitrary streaming ion beam. Additionally, Mahmood and Mushtaq explored the effect of dust streaming on dust acoustic solitary waves in magnetized dusty plasmas, while Gill *et al.* studied dust acoustic solitary waves in a finite-temperature dusty plasma with variable dust charge.

Aleksander Drenik and Richard Clergereaux also investigated the deposition of nanocomposite thin films in dusty plasma. Merlino *et al.* studied nonlinear dust acoustic shocks and structures within dusty plasma.

In this paper, we focus on dust ion acoustic waves in dusty plasma, which consists of negatively charged dust particles, electrons, and non-thermally distributed ions. We derive the stationary solution of the KdV equation using the reductive perturbative method.

Basic Equations

We consider the propagation of dust ion-acoustic waves in a warm plasma consisting of positive and negative dust ions and the usual electrons. The fluid equation of motion governing the collision of less warm plasma is-

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial v_i}{\partial t} + V_i \frac{\partial v_i}{\partial x} + \frac{\sigma}{n_i}\frac{\partial p_i}{\partial x} = -Z_d \frac{\partial \phi}{\partial x} \qquad (2)$$

$$\frac{\partial p_i}{\partial t} + V_i \frac{\partial p_i}{\partial x} + 3P_i \frac{\partial v_i}{\partial x} = 0 \qquad (3)$$

for the positive ions,

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_d)}{\partial x} = 0 \qquad (4)$$

$$\frac{\partial v_d}{\partial t} + V_d \frac{\partial v_d}{\partial x} + \frac{\sigma}{Q n_d}\frac{\partial p_d}{\partial x} = \frac{Z_d}{Q} \frac{\partial \phi}{\partial x} \qquad (5)$$

$$\frac{\partial p_d}{\partial t} + V_d \frac{\partial p_d}{\partial x} + 3P_d \frac{\partial v_d}{\partial x} = 0 \qquad (6)$$

for the negative dust ions,

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} = 0 \qquad (7)$$

$$\frac{\partial n_e}{\partial t} + V_e \frac{\partial n_e}{\partial x} + \frac{1}{Q n_e} \frac{\partial p_e}{\partial x} = \frac{z_d}{Q'} \frac{\partial \emptyset}{\partial x} \quad (8)$$

$$\frac{\partial p_e}{\partial t} + V_e \frac{\partial p_e}{\partial x} + 3 P_e \frac{\partial v_e}{\partial x} = 0 \quad (9)$$

for the isothermal electron,
Again, for the charge imbalance, these equations are to be combined with the Poisson equation

$$\frac{\partial^2 \emptyset}{\partial x^2} = n_e + z_d n_d - n_i \quad (10)$$

where the suffixes i, d, and e stand for positive ion, negative dust ion and electron respectively, $Q' = m_d/m_i$ (dust ion to ion mass ratio), $Q' = m_e/m_i$ (electron to ion mass ratio) and $\sigma = T_i/T_e$ is the ratio of the ion temperature ($T_i = T_d$) to the electron temperature. $z_{d,}$ number of charges on dust particles. The physical quantities appearing in these equations are normalized as follows the densities. n_i, n_d, and n_e by unperturbed electron density n_{eo}, ion pressure p_i, p_d by characteristic ion pressure $k_B n_{eo} T_d$, time 't' by the inverse of the characteristic ton plasma frequency $w_{pi}{}^{-1} = ({}^{m_i}/{}_{4\pi n_{eo} e^2})^{1/2}$, distance 'x' by Debye length $\lambda_{De} = ({}^{k_B T_e}/{}_{4\pi n_{eo} e^2})^{1/2}$, velocities by the ion-acoustic $C_s = ({}^{k_B}/{}_{m_i})^{1/2}$ and electric potential $\emptyset$ by $({}^{k_B T_e}/{}_{e})$; k_B is the Boltzmann constant.

Derivation of KdV Equation and Its Solution

To derive the KdV equation from the basic equations (1)-(10) for the description of the propagation of dust ion-acoustic waves, we expand the flow variables asymptotically about the equilibrium state in terms of the smallness parameter ε as follows:

$$\begin{aligned}
n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \ldots\ldots \\
n_d &= n_{d0} + \varepsilon n_{d1} + \varepsilon^2 n_{d2} + \ldots\ldots \\
n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \ldots\ldots \\
v_i &= \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \ldots\ldots \\
v_d &= \varepsilon v_{d1} + \varepsilon^2 v_{d2} + \ldots\ldots \\
v_e &= \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \ldots\ldots \\
p_i &= p_{i0} + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \ldots\ldots \\
p_d &= p_{d0} + \varepsilon p_{d1} + \varepsilon^2 p_{d2} + \ldots\ldots \\
p_e &= p_{e0} + \varepsilon p_{e1} + \varepsilon^2 p_{e2} + \ldots\ldots \\
\emptyset &= \varepsilon \emptyset_1 + \varepsilon^2 \emptyset_2 + \ldots\ldots
\end{aligned} \quad (11)$$

Again, for the derivation of the nonlinear equation governing the nonlinear dynamics of the wave, we make the usual stretching of the space co-ordinate and time by

$$\xi = \varepsilon^{1/2}(X - Vt), \text{ and } \tau = \varepsilon^{3/2} V t \quad (12)$$

Where V is the linear phase velocity and ε is a smallness parameter measuring the dispersion and nonlinear effects. Using the equation (11) and (12) in the equations (1) ------(10) and equating the co-efficient of the lowest order perturbation in ε with the use of the boundary conditions n_{i1}= n_{d1}=0,
v_{i1}=v_{d1}=0, $\emptyset_1$=0, at $|\xi| \to \infty$ we get

$$\begin{aligned}
n_{i1} &= \frac{z_d n_{io} \emptyset_1}{V^2 - 3\frac{p_{io}\sigma}{n_{io}}}, & n_{d1} &= \frac{-z_d n_{do} \emptyset_1}{QV^2 - 3\frac{p_{do}\sigma}{n_{do}}} \\
n_{e1} &= \frac{-z_d \emptyset_1}{Q(V^2 - 3\frac{p_{eo}}{Q})}, & v_{i1} &= \frac{z_d V \emptyset_1}{V^2 - 3\frac{p_{io}\sigma}{n_{io}}} \\
v_{d1} &= \frac{-z_d V \emptyset_1}{QV^2 - 3\frac{p_{do}\sigma}{n_{do}}}, & v_{e1} &= \frac{-z_d V \emptyset_1}{Q(V^2 - 3\frac{p_{eo}}{Q})} \\
p_{i1} &= \frac{3 p_{io} z_d \emptyset_1}{V^2 - 3\frac{p_{io}\sigma}{n_{io}}}, & p_{d1} &= \frac{-3 p_{do} z_d \emptyset_1}{QV^2 - 3\frac{p_{do}\sigma}{n_{do}}}
\end{aligned} \quad (13)$$

$p_{e1} = \frac{-3p_{i0} z_d \emptyset_1}{Q(V^2 - 3\frac{p_{e0}{}^{\sigma}}{Q})}$

And $1 + z_d n_{d0} - n_{i0} = 0$ $n_{e1} + z_d n_{d1} - n_{i1} = 0$

Now, using $p_{i0} = n_{i0}$, $p_{d0} = n_{d0}$, $p_{e0} = p_{e0} = 1$ in the last equation of (13) i.e. $n_{e1} + z_d n_{d1} - n_{i1} = 0$

The expression for the ion-acoustic speed V can be written as

$$\frac{1}{Q'(V^2 - \frac{3}{Q})} + \frac{z_d n_{do}}{QV^2 - 3\sigma} + \frac{n_{io}}{V^2 - 3\sigma} = 0 \quad (14)$$

Equation (14) is a quadratic equation in V^2, and consequently, it represents two types of ion-acoustic modes propagating with different phases, namely

$$V_1{}^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (15)$$

For the fast ion-acoustic mode

and $V_2{}^2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ (16)

for the slow ion-acoustic mode

where, $a = Q + \frac{Q'}{1 - rz_d}(z_d r + Q)$

$$b = 3\sigma\left(1 + Q + \frac{Q' r z_d}{1 - rz_d} + \frac{Q'}{1 - rz_d}\right) + \frac{3Q' r z_d}{Q(1 - rz_d)} + \frac{3Q'}{1 - rz_d} \quad (17)$$

$$c = 9\sigma\{\sigma + \frac{Q'(1 + rz_d)}{Q(1 - rz_d)}\}$$

And $r = \frac{n_{do}}{n_{io}}$, $n_{i0} = \frac{1}{1 - rz_d}$, $n_{d0} = \frac{r}{1 - rz_d}$

Again, equating the co-efficient of the next order perturbation, we can get the following equations:

$$V\frac{\partial n_{i1}}{\partial \tau} + v_{i1}\frac{\partial n_{i1}}{\partial \xi} - V\frac{\partial n_{i2}}{\partial \xi} + n_{i1}\frac{\partial v_{i1}}{\partial \xi} + n_{i0}\frac{\partial v_{i2}}{\partial \xi} = 0 \quad (18)$$

$$V\frac{\partial n_{d1}}{\partial \tau} + v_{d1}\frac{\partial n_{d1}}{\partial \xi} - V\frac{\partial n_{d2}}{\partial \xi} + n_{d1}\frac{\partial v_{d1}}{\partial \xi} + n_{d0}\frac{\partial v_{d2}}{\partial \xi} = 0 \quad (19)$$

$$V\frac{\partial n_{d1}}{\partial \tau} + v_{d1}\frac{\partial n_{d1}}{\partial \xi} - V\frac{\partial n_{d2}}{\partial \xi} + n_{d1}\frac{\partial v_{d1}}{\partial \xi} + n_{d0}\frac{\partial v_{d2}}{\partial \xi} = 0 \quad (20)$$

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (21)$$

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (22)$$

$$V\frac{\partial n_{e1}}{\partial \tau} - Vn_{e1}\frac{\partial v_{e1}}{\partial \xi} - V\frac{\partial v_{e2}}{\partial \xi} + v_{e1}\frac{\partial v_{e1}}{\partial \xi} + \frac{1}{Q}\frac{\partial p_{e2}}{\partial \xi} = \frac{z_d}{Q'}n_{e1}\frac{\partial \emptyset_1}{\partial \xi} + \frac{z_d}{Q'}n_{d0}\frac{\partial \emptyset_2}{\partial \xi} \quad (23)$$

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (24)$$

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (25)$$

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (26)$$

$$\frac{\partial^2 \emptyset_1}{\partial \xi^2} = n_{e2} + z_d n_{d2} - n_{i2} \quad (27)$$

Now eliminating $\frac{\partial v_{i2}}{\partial \xi}$ from equations (18) and (21), $\frac{\partial v_{d2}}{\partial \xi}$ from equations (19) and (22), $\frac{\partial v_{e2}}{\partial \xi}$ from equations (20) and (23) respectively, we get

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (28$$

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (29)$$

$$Vn_{i0}\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \xi} - Vn_{i0}\frac{\partial v_{i2}}{\partial \xi} + v_{i1}n_{i0}\frac{\partial v_{i1}}{\partial \xi} + \sigma\frac{\partial p_{i2}}{\partial \xi} += -z_d n_{i1}\frac{\partial \emptyset_1}{\partial \xi} - z_d n_{i0}\frac{\partial \emptyset_2}{\partial \xi} \quad (30)$$

Again eliminating $\frac{\partial p_{i2}}{\partial \xi}$ from the equation (24) and (28), $\frac{\partial p_{d2}}{\partial \xi}$
From equations (25) and (29), we get respectively.

$$\frac{\partial n_{i2}}{\partial \xi} + \frac{2V^2 z_d n_{i0}}{(V^2-3\sigma)^2}\frac{\partial \emptyset_1}{\partial \tau} + \frac{3(V^2-2\sigma+3)}{(V^2-3\sigma)^3} z_d{}^2 n_{i0}\emptyset_1 \frac{\partial \emptyset_1}{\partial \xi} + \frac{z_d n_{i0}}{(V^2-3\sigma)}\frac{\partial \emptyset_2}{\partial \xi} = 0 \qquad (31)$$

$$-\frac{\partial n_{d2}}{\partial \xi} + \frac{2V^2 Q z_d n_{d0}}{(QV^2-3\sigma)^2}\frac{\partial \emptyset_1}{\partial \tau} + \frac{3(QV^2+\sigma)}{(QV^2-3\sigma)^3} z_d{}^2 \emptyset_1 n_{d0}\frac{\partial \emptyset_1}{\partial \xi} - \frac{z_d n_{d0}}{(QV^2-3\sigma)}\frac{\partial \emptyset_2}{\partial \xi} = 0 \qquad (32)$$

The values of n_{i1}, v_{i1}, $n_{d1}, v_{d1}, p_{i1}, p_{d1}, v_{e1}, n_{e1}$ are submitted from (13) in deducing these equations
Adding the equations (30), (31), (32) and making use of the relations (27) and (14), the KdV equation can be obtained as

$$\frac{\partial \emptyset_1}{\partial \tau} - p\emptyset_1 \frac{\partial \emptyset_1}{\partial \xi} - q\frac{\partial^3 \emptyset_1}{\partial \xi^3} \qquad (33)$$

Where, $p = AB$, $q = A/2$

$$A = \frac{(V^2-3\sigma)^2(QV^2-3\sigma)^2 Q'(QV^2-3)^2}{V^2 z_d[(QV^2-3\sigma)^2 Q'(QV^2-3)^2 + Qz_d n_{do}(V^2-3\sigma)^2 Q'(QV^2-3)^2 + Q^2(QV^2-3\sigma)^2(QV^2-3\sigma)^2]} \qquad (34)$$

$$B = \frac{1}{2}\left[\frac{(3V^2-6\sigma+9)n_{io}}{(V^2-3\sigma)^3} - \frac{3(QV^2+\sigma)z_d n_{io}}{(QV^2-3\sigma)^3} - \frac{3Q^2(QV^2+1)}{Q'^2(QV^2-3\sigma)^3}\right]z_d{}^2 \qquad (35)$$

Ion acoustic solitons do not exist for $V^2 = 3\sigma$, $3\sigma/Q$ and $3\sigma/Q$

Introducing the variable $\eta = \xi - c\tau$ and using the boundary conditions

$$\emptyset_1(\eta) = \frac{d\emptyset_1}{d\eta} = \frac{d^2\emptyset_1}{d\eta^2} = 0$$

As $|\eta| \to \infty$, We get the solution of the KdV equation (33) as

$$\emptyset_1 = \emptyset_0 sech^2 \frac{\eta}{\Delta} \qquad (36)$$

Where $\emptyset_0 = \frac{3c}{p}$ is the amplitude of the ion acoustic solutions, $\Delta = \left(\frac{4q}{c}\right)^{1/2}$ is the width of the ion-acoustic solitons and $c = V - 1$ is the excess (but small) of the value of V given by equations (15)-(17) over one for a small amplitude limit.

The amplitudes and widths of the solitons depicted in the figures are computed for varying values of c (but not for particular values of c) for each triplet assigned values of Q, σ, and r. The consideration of electron inertia $Q'(= m_e/m_i)$ and its implicit occurrence in the second order equations in a neglecting those of ε^3 ascertains that $\varepsilon \sim Q'$ $n > 1$ slightly [27, 28], so that $\varepsilon < Q'$. This indicates that the effects on the measurable quantities of solitons that is on solitons amplitudes and widths are countable to the order equal or even higher than Q'. In the composition of the plasma in our consideration, we assume $Q' = .54 \times 10^{-3}$ concerning the lightest ion in the subsequent discussion.

Discussion

The nonlinear wave structure of small-amplitude dust ion-acoustic solitary waves is studied in a warm dusty plasma with consideration of electron inertia. Notably, depending on various mass ratio values Q, temperature ratio σ, and density ratio r, only rarefactive solitons are present in this plasma model. This finding represents a novel outcome of the investigation. The presence of warm dust ions is primarily responsible for the exclusivity of rarefactive solitons in this system.

As shown in Figure 1, the amplitude of the rarefactive dust ion-acoustic solitons increases exponentially with an increase in dust charges, while keeping. $Q = 0.002$, $r = 0.05$ fixed for different values of the temperature ratio σ Furthermore, the amplitudes of the rarefactive solitons are higher when the temperature ratio decreases, particularly for smaller values.

Regarding the temperature ratio, the amplitude of solitons is higher at lower temperatures. Conversely, the width (A) of rarefactive solitons rapidly decreases (as shown in Figure 2) for a fixed value of Q = 0.002 and r = 0.05, while varying the values of σ. The amplitude of rarefactive solitons increases linearly as the number of dust charges increases (illustrated in Figure 3) for fixed r = 0.2, σ = 0.025, and different Q values. It is observed that the soliton amplitude is greater for high values of Q, given that the values of z_d, r and σ remain constant, compared to low values of Q. Additionally, the amplitude of rarefactive solitons increases as the density ratio rises (depicted in Figure 4) for a fixed Q = 0.001, σ = 0.2, and varying values of z_d. For high values of z_d, the amplitude increases significantly as the density ratio goes up.

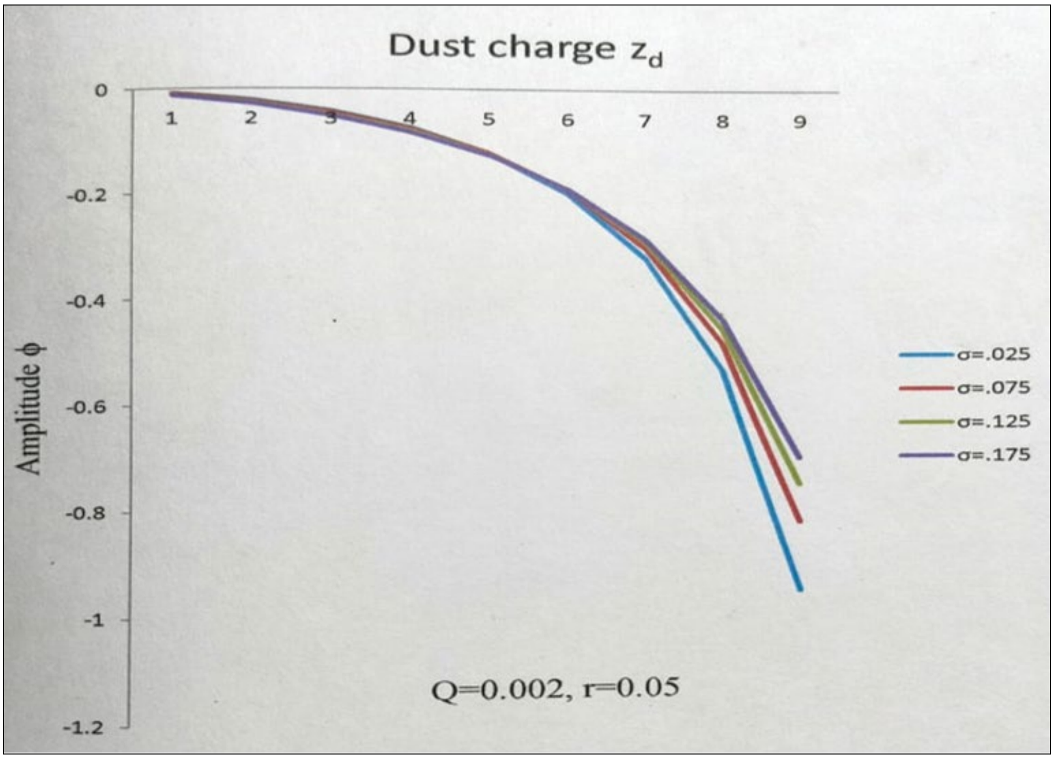

Fig 1: Amplitude $(\emptyset)$ of dust ion-acoustic solitons versus dust charge (z_d) for fixed $Q = .002$ and $r = 0.05$ for different values of $\sigma = 0.025,\ 0.075,\ 0.125,\ 0.175,\ 0.2$

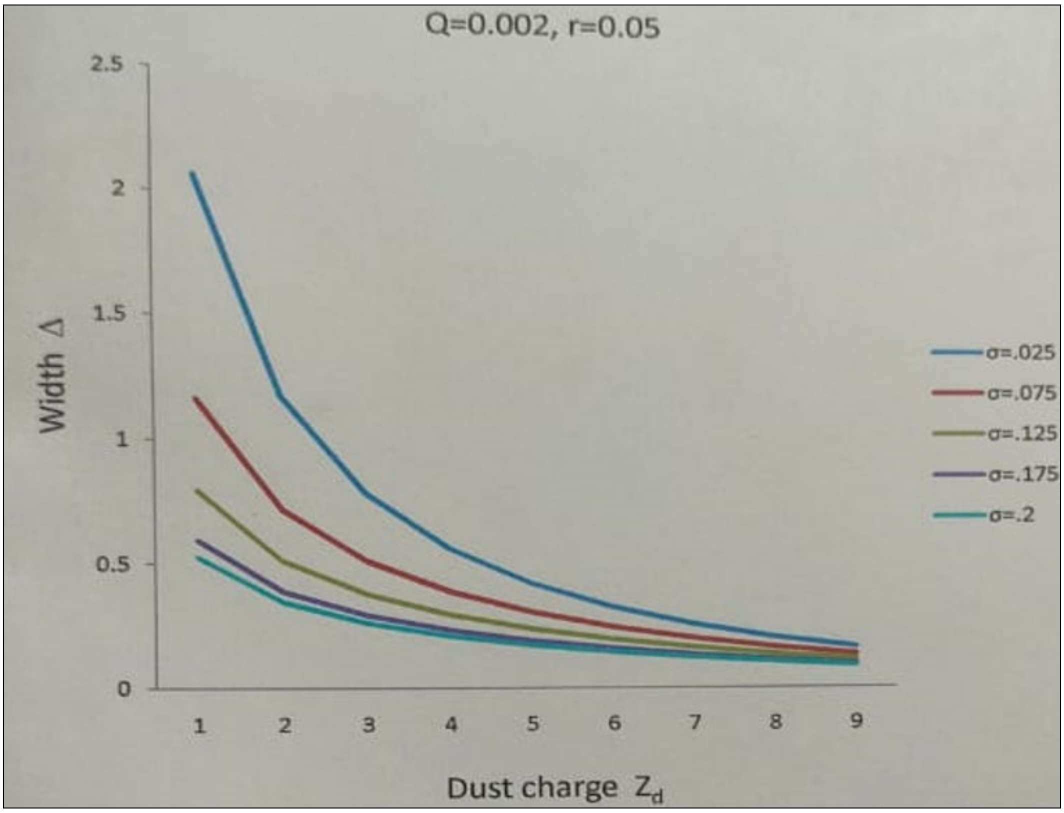

Fig 2: Width (Δ) of rarefactive dust ion-acoustic solitons versus dust charge (z_d) for fixed $Q = .002$ and $r = 0.05$ for different values of $\sigma = 0.025,\ 0.075,\ 0.125,\ 0.175,\ 0.2$

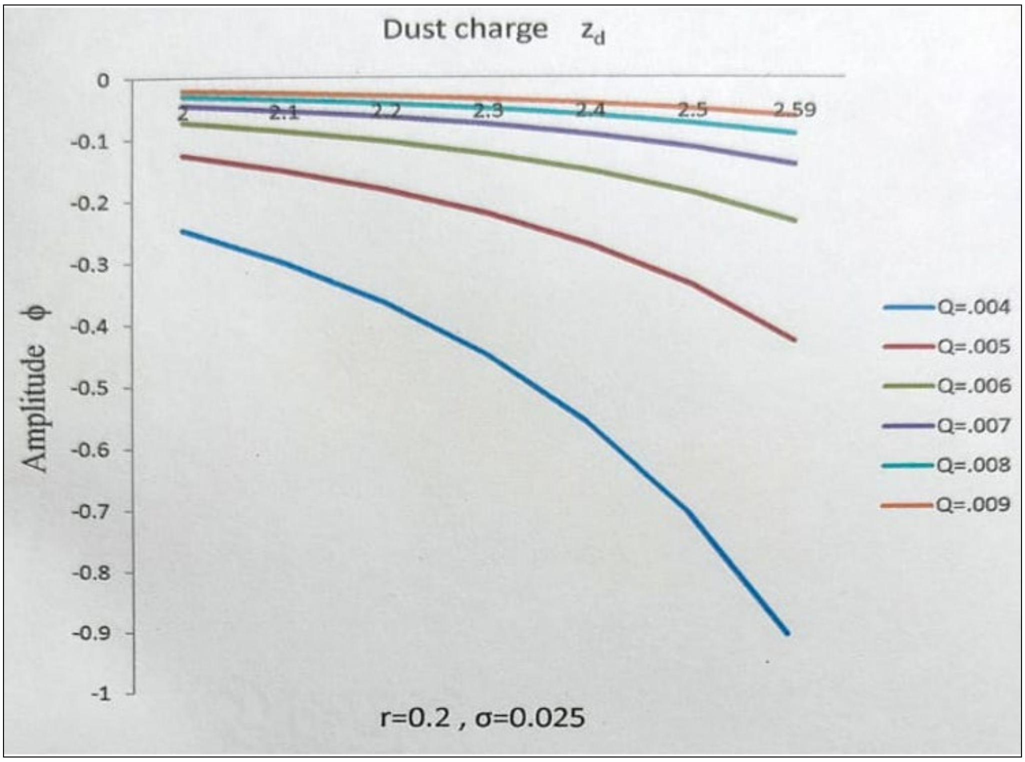

Fig 3: Amplitude $(\emptyset)$ of rarefactive ion-acoustic solitons versus dust charge z_d for fixed $(r) = 0.2$ and $\sigma = 0.025$ for different values $Q = .004, .005, .006, .007, .008, .009$

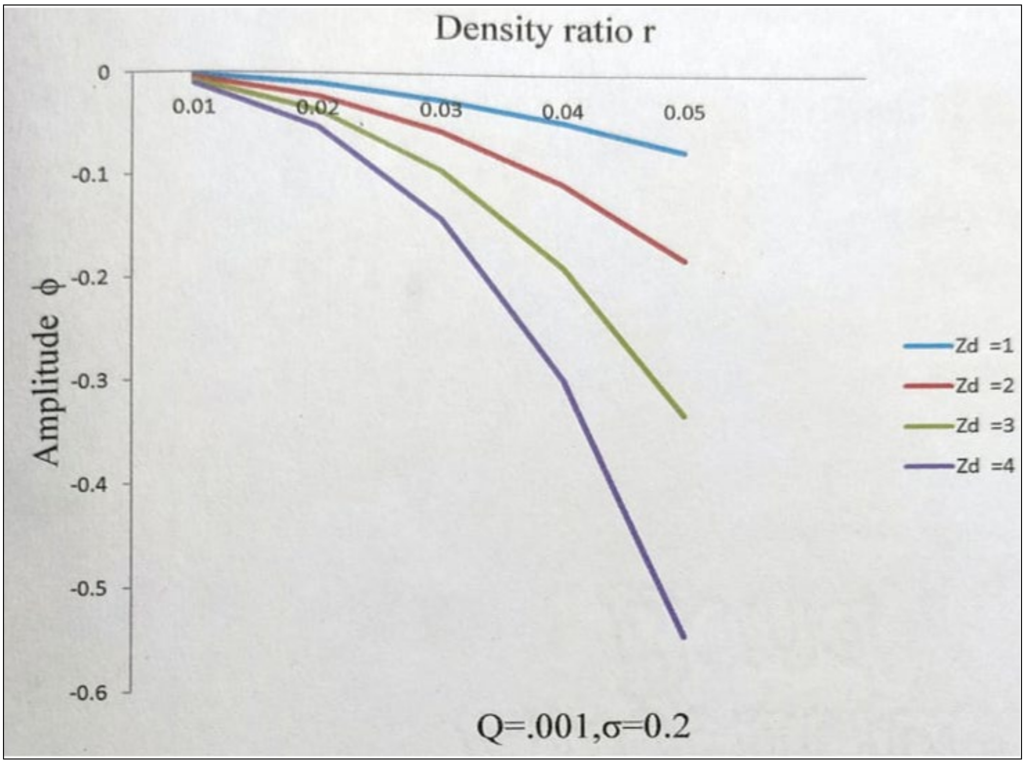

Fig 4: Amplitude of rarefactive ion-acoustic solitons versus density ratio (r) for fixed $Q = .001$ and $\sigma = .2$ for different values of $z_d = 1, 2, 3, 4$

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