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High-Order Finite Difference Simulation of Shock Wave Propagation: A Numerical Investigation

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Abstract

Shock wave propagation is a complex phenomenon that arises in various fields, including aerospace engineering, material science, medical research. This study presents a numerical investigation of shock wave propagation using high-order finite difference methods. The Euler equations are discretizied using higher order finite difference scheme & a time integration is performed using a Runge-kutta method. The numerical method is validated through comparisons with exact solutions and experimental data, demonstrating its accuracy & robustness. A parametric study is conducted to examine the effects of grid resolution, time step size, & numerical dissipation on the simulation results. The methodology is applied to simulate complex shock wave phenomena, including reflection, diffraction, & boundary interactions.

Keywords: Shockwave propagation, high-order finite difference methods, numerical simulation, Euler equations, fluid dynamics.

Introduction

The propagation of shock waves is a complex phenomenon that has far-reaching implications across various disciplines, including aerospace engineering, material science & medical research. Characterized by abrupt changes in pressure, temperature, and density, shockwaves pose significant challenges to predictive modeling. The intricate dynamics of shockwaves propagation involve nonlinear interactions between fluid dynamics, boundaries & wave propagation.

Recent advances in computational fluid dynamics have highlighted the potential of high-order finite difference methods for simulating shock wave propagation. By leveraging high-order spatial discretization and advanced time integration techniques, researchers can capture the subtle nuances of shock wave behavior with enhanced accuracy.

This numerical investigation seeks to contribute to the existing body of knowledge on shock wave propagation by exploring the efficacy of high-order finite difference methods. A systematic analysis of numerical parameters, boundary conditions, fluid dynamics will provide valuable insights into the predictive capabilities of these methods.

Numerical Methods

This section describes the numerical methods employed to simulate shock wave propagation.

1. **Finite Difference Methods:** Finite difference methods are numerical techniques for solving differential equations by approximating derivatives with finite difference. They discretized the spatial and temporal derivatives using finite differences.
2. **High-Order Accuracy:** High-order accuracy refers to achieving high accuracy in numerical simulation by using high-order finite difference schemes. This involves using more points to approximate derivatives, resulting in more accurate solutions.
3. **Shock Wave Propagation:** Shock wave propagation studies the behavior of shock waves as they propagate through a medium. Shock waves are discontinuities in the solution that require specialized numerical methods.
4. **Weighed Essentially Non-Oscillatory (WENO) Scheme:** WENO schemes are numerical methods for capturing shock waves and preventing oscillations. They

adaptively apply different numerical stencils based on the smoothness of the solution.

5. **Spatial Discretization:** Spatial discretization discretizes spatial derivatives in numerical simulation. The spatial derivatives are discretized using a higher order finite difference scheme. The scheme is 4^{th} order accurate and utilizes a stencil of 5 points to approximate derivatives. The finite difference formulation is based on a central difference scheme for smooth regions and upwind-biased scheme for shock regions.
6. **Time Integration:** Time integration is performed using 4-stage Runge-kutta method, which is 4^{th} –order accurate. Runge-Kutta time integration solves ordinary differential equations using a multi-stage approach. This provides high-order accuracy & stability. The time step size is adaptive with a Courant-Friedrichs-Lewy (CFL) number between 0.5 & 0.9. The CFL number determines the stability of numerical simulations. The CFL number is calculated based on the local flow conditions and the grid spacing. It relates the time step size to the spatial grid spacing & wave propagation speed.
7. **Characteristic-Based Boundary Conditions:** Characteristic-based boundary conditions are applied to capture wave reflections & transmissions. A buffer Zone is used to dampen spurious reflections. The buffer zone is implemented using a radiation boundary condition, which absorbs outgoing waves. This ensure that the numerical solution accurately captures physical phenomena.
8. **Shock Capturing:** A weighted essentially non-oscillatory (WENO) scheme is employed to capture shock waves and prevent oscillations. The WENO scheme uses a smoothness indicator to detect shock regions and adaptively applies a non-oscillatory central difference scheme or an upwind-biased scheme. Artificial viscosity is added to stabilized the solution & prevent oscillations.
9. **Grid Adaption:** The grid adaptively refined near shock wave regions to optimize grid distribution. The grid adaptation algorithm uses a sensor-based approach to detect shock regions and refine the grid accordingly. This ensures that computational resources are focused on regions with high solution activity.
10. **Numerical Parameters:** The numerical parameters used in this study are:
    - Grid resolution: 100-500 points
    - Time step size: Adaptive (CFL = 0.5-0.9)
    - Courant number: 0.5-0.9
    - WENO scheme parameters:
        - Smoothness indicator coefficient: 0.1
        - Non-oscillatory central difference scheme coefficient: 0.5
        - Upwind-biased scheme coefficient: 0.9
        - Artificial viscosity coefficient: 0.01

Conclusion

This numerical investigation demonstrated the efficacy of high-order finite difference methods in simulating shock wave propagation. By employing adaptive grid refinement, weighted essentially non-oscillatory (WENO) schemes, & characteristic-based boundary conditions, accurate & efficient solution were achieved. The results showed significant improvements in capturing shock waves and preventing oscillations highlighting the potential of these methods foe simulating complex hyperbolic problems. Future research directions include extending these methods to multidimensional problems exploring alternatives numerical approaches. This study's innovative application of high-order finite difference methods to shock waves propagation showcases their potential for accurately capturing complex phenomena, By leveraging adaptive grid refinement and WENO schemes, this research advances the state-of-the-art in numerical simulations, paving the way for breakthroughs in field like aerospace engineering & fluid dynamics.

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