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Connection and Convariant Differentiations in f_n and Curvature Tensors in F_n

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Abstract
Takano, K[5] has studied the existence of existence of affine motion in a non –Riemannian K*-space has obtained several result of significance. He has also studied the existence of projective motion in a Riemannian space with bi-recurrent curvature. Pande, H.D. and Kumar. A[3] have also discussed the special infinitesimal projective transformation in a Finsler space and have obtained certain theorms therien . In the present communication we have also derived the complete of a affine motion in a R+ - recurrent Finsler space. We have also derived the complete condition for the vanishing of $£_v\, R\, jkh$ is the curvature tensor type entity with respect to the non- symmetric connection as has been introduced by Gupta [2].

Keywords: Transformation, Orthogonality, Hamiltonian Function, n Dimension, Positional Co-ordinate, Directional Co-ordinate, Minkowskin Space.

1. Introduction

Consider a region R of an n-dimension space X_n which is completely covered by a coordinate system, such that each point P in R in represented by n-tuples $x^1, x^2, x^3, \dots \dots x^n$ *or* x^i $(i = 1,2,3 \dots . . n)$ of real number, Which will be called coordinates are function of a single parameter 't' (say). Thus, the parametric equations

(1.1.) $x^i = x^i\,(t)$ represent a curve C of X_n

Let us assume that this curve is of class C1. the entities (derivatives)

(1.2) $xi = dx^i/dt$

constitute the components of a vector tangential to the curve C at the point $P(x^i)$. The combination $(x^i,\ x^i)$ which is conveniently weitten as $(x,\ \dot{x})$ is known as line-element of the curve C with centre at P[28]. In the line element $(x, \dot{x})$ x and $\dot{x}$ are called positionl and directionl coordinates respectively.

Let $P(x)$ and $Q(x + dx)$ be two neigbouring points of the region R then the infinitesimal distance ds between these two points P and Q is defined by

(1.3) $ds\ F(x, dx)$

where
$F\ (x, \dot{x})$ is a function defined for all line - elements $(x, \dot{x})$ in the region R.

Defenition (1.1)
An η– dimensional space X_n equipped with a function $F(x, \dot{x})$ as defined above, denoted by F_n, is called a Finsler space if the function F statisfies the following three conditions [28]:

Condition-A
The function $F(x, \dot{x})$ is positively homogrneous of degree one in X^i, i.e. where k is some positive scalar.

Condition-B

The function $F(x, x^{\cdot})$ is positive unless all $x^{\cdot i}$s vanis simultaneously, i.e. (1.5) $F(x, x^{\cdot}) > 0$ with $\sum_i (x\iota^{\cdot})^2 \neq 0$.

Condition-C

The quadratic form

(1.16) $\{\partial\iota^{\cdot}, \partial j\, F^2 (x, x^{\cdot})\}$ $(i,$ (j where $\partial\iota^{\cdot} = \partial/{}^{\cdot}\iota\, \partial x$ is positive definite for all the variables (i

The function F$(x, x^{\cdot})$ is called as the fundamental function or the metric function of the Finsler space F_n.

2. Metric Tensor

Let us consider a set of quantities g_{ij} which are defined by

(2.1) $g_{ij}(x, x^{\cdot})\, \frac{1}{2}\, \partial^{\iota^{\cdot}}\, \partial^{y^{\cdot}}\, F2\,(x, x^{\cdot})$

The quantities g_{ij} countitute the components of a covariant tensor of type (0.2). This tensor is called metric tensor of the Finsler space F_n. From (2.1) it is obvious that the metric tensor $g_{ij}(x, x^{\cdot})$ are positively homogeneous of degree zero in $x^{\cdot i}$ and symmetric in i and j. In view of Euler's theorm on homogeneous function, we have

(2.2) (a) $x^{\cdot i}\partial_{\iota^{\cdot}}\, F(x, x^{\cdot}) = {}^{\cdot}F\,(x, x^{\cdot})$,

(b) $x^{\cdot i}\partial_{\iota^{\cdot}}\, \partial_{y}^{\cdot}\, F\,(x^i, x^{\cdot\iota}) = 0$

(c) $det.\, \{\partial_{\iota}^{\cdot}\, \partial_{y}^{\cdot}\, F\,(x^i, x^{y^{\cdot}})\} = 0$ and

(d) $g_{ij}(x, x^{\cdot})x^{\cdot\iota}x = F^2(x, x^{\cdot}) = 0$

In view of (2.2d), we may express the infinitesimal distance ds between two neighboring points x and $x + dx$ in tearms of metric tensor as

(2.3) $ds^2 = g_{ij}\,(x, dx)dx^i\, dx^j$

3. Tangent Space, Minkowskian Space, Indicatrix and its Dual Space

Let us consider the change of local coordinates represented by

(3.1) $\overline{x^i} = \overline{x^i}\,(x^j\,(t))$

Then the components $x^{\;i^{\cdot}} = /dt$ of the tangent vector to the curve (1.1) are transformed according to

(3.2) $x^{\cdot\bar{i}} = (\partial_j^{\cdot}\, \overline{x^i})x^{\cdot\bar{J}}\, \partial_j = \partial/\partial x^i$

or in terms of differentials

(3.3) $d\overline{x^i} = (\partial^j\overline{x^i})d\overline{x^i}$

A system of n-quantities X^i whose transformation law under (3.1) is analogous to that of the $x^{\cdot i}$ is called a contravariant vector attached to the point $P(x^i)$ of F_n. The individual X^i represents its components. Such contravarient vectors attached to the point $P(x^i)$ constitute the elements of a vector space and this vector space is said to be the tangent space attached to the point $P(x^i)$ and is represented by T_n (P) or T_n (x^i).

This may be defined in terms of differentials as well. The length of a vector ni of a vector η^i $T_N(P)$ is given by $F(x^i, \eta^i)$. In view of (2.2d), all length in $T_N(P)$ may be expressed in terms of g_{ij}, defined by (2.1), which we shall regard as the components of the metric tensor of $T_N(P)$.

Definitaion (3.1)

A Finsler space F_N Is called Minkowskian space if there exists a coordinate system in which the metric function F is independent of x^i ([28],p.50)

Definition (3.2)

The indicatrix of F_n at a Point x^i is defined by the equation $F(x, x)$ =1(x^i is fixed) (16, 28, p. 12).

Definition (3.3)

A tensor T of F_N is called an indicatory ([17]) if its components $T_{ij}...k$ satisfy

(3.4) $T_{oj\ldots..k} = T_{io\ldots..}k = \cdots = 0$,

Where 0 denoted the contraction with xi

Corresponding to each aebitrary contravariant vector (x^i of Tn (p), there is a covariant vector y^i, such that

(3.5) $y^i = g_{ij}(x, x^{\cdot})x^{\cdot j}$

All such of vector associate with the point P of F_n, constitute a vector space function of the which is named as dual tangent space at P and is represented by

$\overline{T_n}(P)$ or $T'_n(P)$. The metric dual tangent space is the Hamiltonian function $H(x^i, x^{\cdot i})$ satisfying the three requisite conditions for beang a Finsler space as have been stated in section 1.

Analogous to the metric tensor $g_{ij}(x,x^{\cdot})x^{\cdot j}$, we define a tensor $g_{ij}(x^k, y_k)$ as

(3.6) $g_{ij}(x^k, y_k) = \frac{1}{2)}\, \bar{\partial}_i\,, \bar{\partial}_{jk}\, H2(x^k, y_k)'$

Where ∂_i denotes the partial differentiation with respect to the covariant vector $x^{\cdot j}$

These quantities $g_{ij}(x^k, y_k)$ constitute the components of a contravariant tensor of the type (2,0).

The quantities g_{ij} defined by (2.1) and (3.6) are related by

$k = \{1\; if\; i=k$

(3.6) $g_{ij}g^{jk} = \delta$

From here we find (3.7) (a) $g_{ij}g_{ij} = n$

$0\; other\; wise$

Transecting (2.1) by $x^{\cdot j}$ and using (3.5) we get

(3.8) $y_i = g_{ij}\, x^j = \frac{1}{2}\, \partial_{\iota}^{\cdot}\, F^2 = F\partial_{\iota}^{\cdot}\, F$

The vector $x^{\cdot j}$ also satisfies the relations

(3.8) $y_i x^{\cdot j} = F^2$ and (b) $g_{uj} = \partial_{\iota}^{\cdot}\, y_j = \partial_{y}^{\cdot}\, y_i$

4. (h) hv-Torsion Tensor and Generalised Christoffel Symbol

From the metric tensor we construct a new tensor $Cijk$ by differentiating (2.1) partially with respect $x^{\cdot k}$. This new tensor C_{ijk} is defined as

(4.1) $C_{\;ijk} = \frac{1}{2}\, \partial_k^{\;\cdot}\, g_{ij} = \quad \frac{1}{2}\, \partial_{\iota}^{\cdot}\, \partial_{y}^{\cdot}\, \partial_{k}^{\cdot}\, F2$

This tensor is called the (h) hv-torsion tensor [18]. It is positively homogenouse of degree-1 in $x^{\cdot j}$ and is symmetric in all three of its indices. Because of its homogeneous properties this tensor satisfies the following identities.

(4.2) (a) $C_{ijk}x^{\cdot i} = C_{jki}x^{\cdot}i = Ckijx^{\cdot}i = 0$

(b) $C_{ijk}x^{\cdot i} = C_{kj}x^{\cdot}$ j = 0 and (b) $Ci\ yi = 0$

The tensor C_{ijk} is the associate tensor of $C_{jk}\ y_i$ and is defined by

(4.3) $C_{ijk}\ g^{hj}C_{ijk}$

This tensor is also positively homogeneous of degree-1 in $x^{\cdot}i$ and is symmetric in its lower indices.

Let us the generalized Christoffel symbols of first and second kind, as Riemannian geometry by

(4.4) (a) $\gamma_{ijk}\ \frac{1}{2}\ (\partial_i\, g_{jk} + \partial_k\, g_{ij} - \partial^j\, g_{ki})$ and $\gamma_{hik}\ g^{hj}\gamma_{ijk}$

5. Magnitude of A Vector the Notion of Orthogonality

The metric tensor $g_{ij}(x, x^{\cdot})$ may be used in two different way, in defining the magnitude of a vector and also the angle between two vectors.

Definition (5.1)

Let Vi be a vector then the scalar |V| given

(5.1) $|V|^2 = g_{ij}(x, v)\ V^iV^j$

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